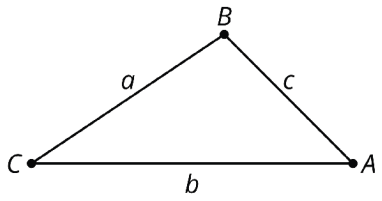
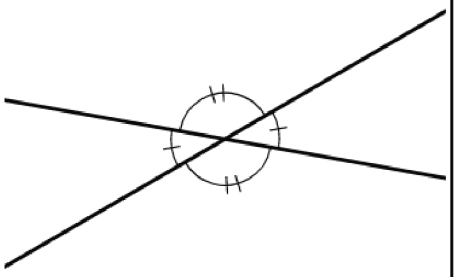
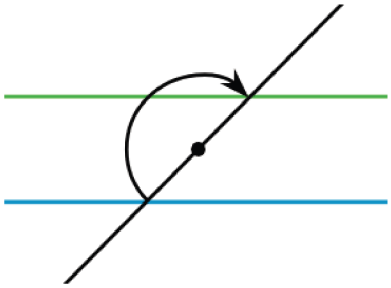
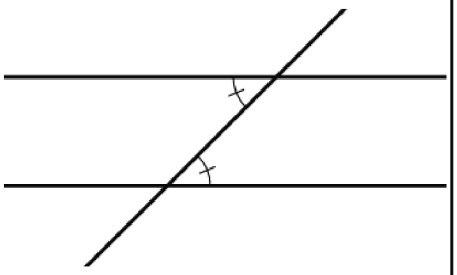
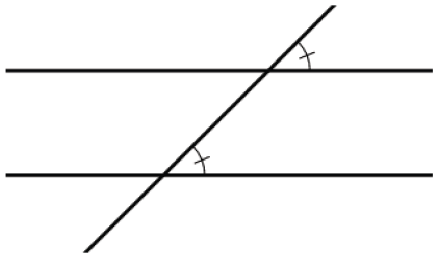
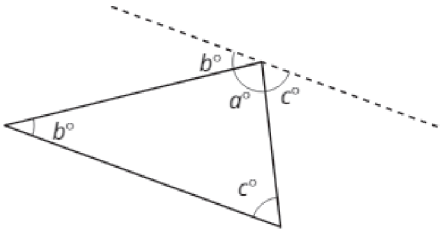
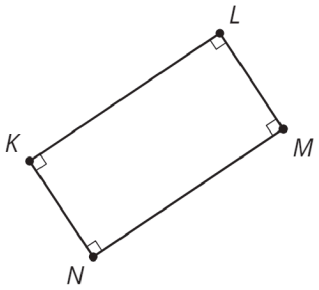
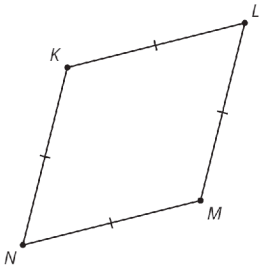
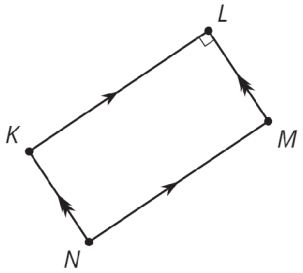
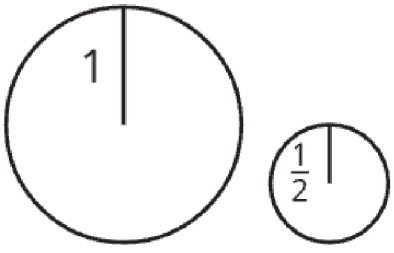
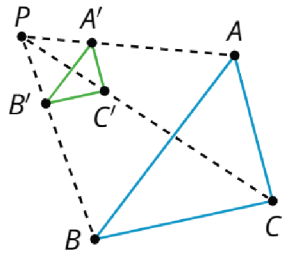
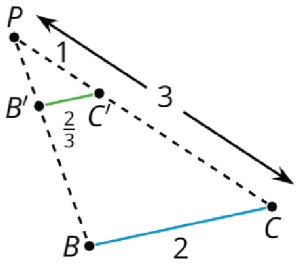
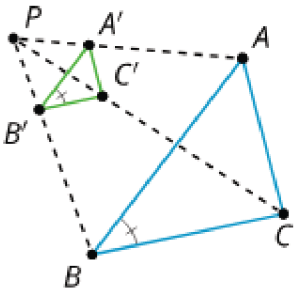
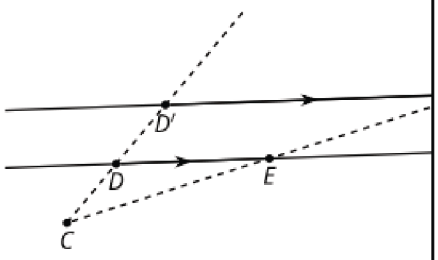
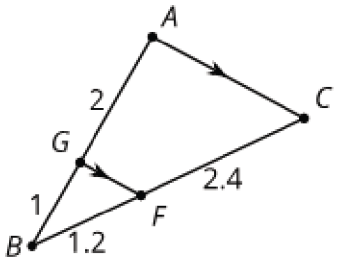
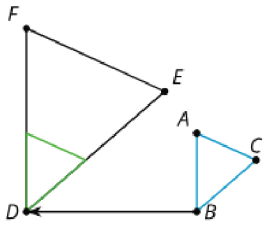
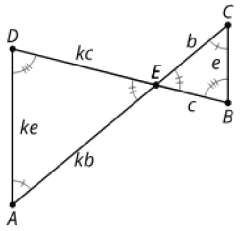
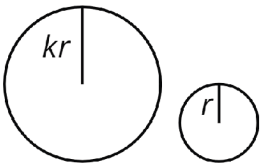
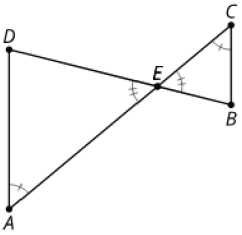
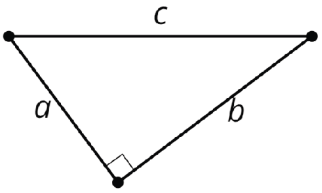
date, type	statement	diagram

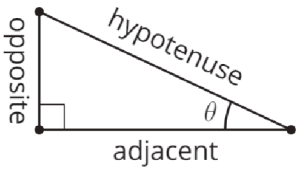
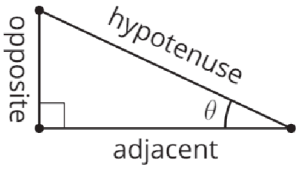
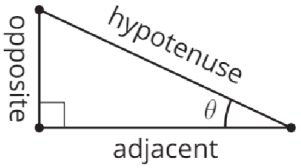
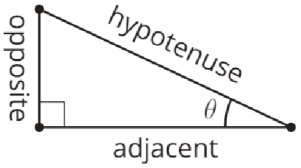
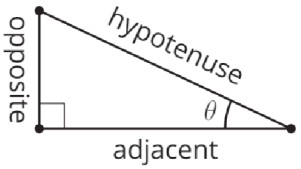
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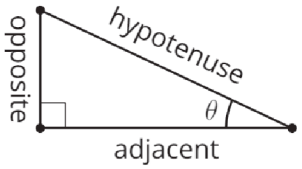
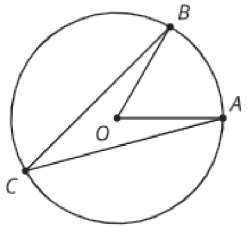
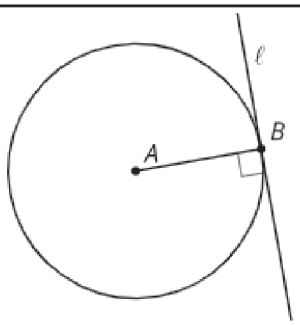
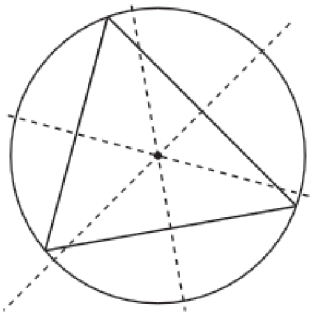
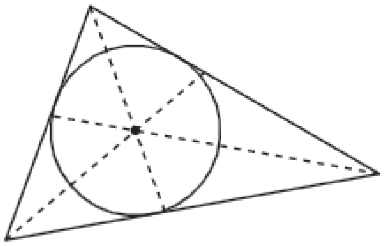
lesson, type	statement	diagram
U1, L3 (students write the date) theorem	**Triangle Inequality Theorem**: If a triangle has side lengths a, b, and c, then $c < a + b$.	
U1, L7 theorem	Vertical angles are congruent.	
U1, L9 assertion	Rotation by 180 degrees takes lines to parallel lines or to themselves.	
U1, L9 theorem	**Alternate Interior Angle Theorem:** If two parallel lines are cut by a transversal, then alternate interior angles are congruent. Conversely, if two lines are cut by a transversal and alternate interior angles are congruent, then the lines have to be parallel.	
U1, L9 theorem	**Corresponding Angle Theorem:** If two parallel lines are cut by a transversal, then corresponding angles are congruent. Conversely, if two lines are cut by a transversal and corresponding angles are congruent, then the lines have to be parallel.	

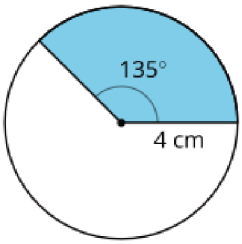
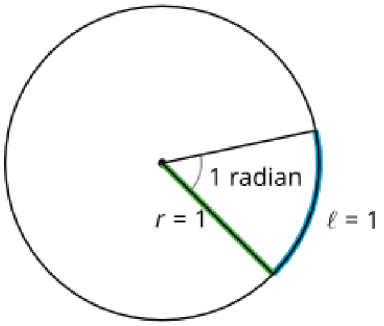
lesson, type	statement	diagram
U1, L10 theorem	**Triangle Angle Sum Theorem:** The three angle measures of any triangle always sum to 180 degrees.	 $a + b + c = 180$
U1, L12 definition	A **rectangle** is a quadrilateral with four right angles.	
U1, L12 definition	A **rhombus** is a quadrilateral with four congruent sides.	
U1, L12 theorem	If a parallelogram has (at least) one right angle, then it is a rectangle.	 *KLMN* has a right angle so it is a rectangle
U2, L1 definition	**Scale factor** is the factor by which every length in an original figure is multiplied when you make a scaled copy.	 Scale factor is 2 or ½

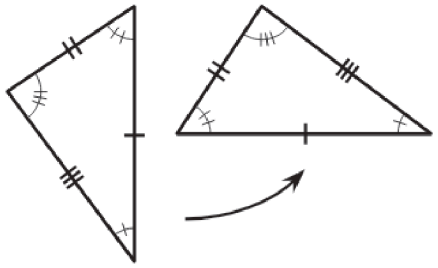
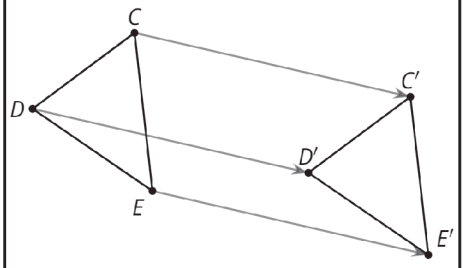
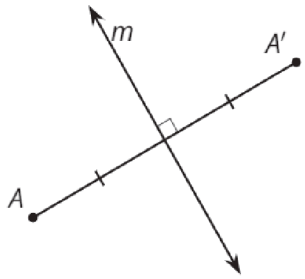
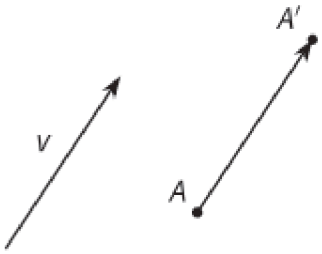
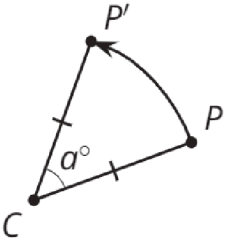
Date, Type	Statement	Diagram
U2, L1 definition	A **dilation** with center P and positive scale factor k takes a point A along the ray PA to another point whose distance is k times farther away from P than A is. Dilate _(object)_ using center _(point)_ and a scale factor of _(number)_.	 $PA' = k \cdot PA$
U2, L3 assertion	The dilation of a line segment is longer or shorter according to the same ratio given by the scale factor.	 $PC:PC' = 3:1$, $BC:B'C' = 2:\frac{2}{3}$
U2, L4 assertion	If a figure is dilated, then corresponding angles are congruent.	 $\triangle A'B'C'$ is a dilation of $\triangle ABC$ so $\angle B \cong \angle B'$
U2, L4 theorem	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	 Dilate using center C. $DE \parallel D'E'$
U2, L5 theorem	If a line divides two sides of a triangle proportionally, the line must be parallel to the third side of the triangle.	 $\frac{1}{2} = \frac{1.2}{2.4}$ so $AC \parallel GF$

Date, Type	Statement	Diagram
U2, L6 definition	One figure is **similar** to another if there is a sequence of rigid motions and dilations that takes the first figure so that it fits exactly over the second.	 Translation and dilation takes $\triangle ABC$ onto $\triangle FDE$ so $\triangle ABC \sim \triangle FDE$
U2, L7 theorem	If two triangles have all pairs of corresponding angles congruent and all pairs of corresponding side lengths in the same proportion, then the two triangles are similar.	 $\angle A \cong \angle C$, $\angle D \cong \angle B$, $\angle DEA \cong \angle BEC$, $\frac{AD}{CB} = \frac{DE}{BE} = \frac{EA}{EC}$ so $\triangle DEA \sim \triangle BEC$
U2, L8 theorem	All circles are similar.	
U2, L9 theorem	**Angle-Angle Triangle Similarity Theorem:** In two triangles, if two pairs of corresponding angles are congruent, then the triangles must be similar.	 $\angle A \cong \angle C$, $\angle DEA \cong \angle BEC$, so $\triangle DEA \sim \triangle BEC$
U2, L16 theorem	**Pythagorean Theorem:** If a right triangle has legs with lengths a and b and hypotenuse with length c, then $a^2 + b^2 = c^2$.	 $a^2 + b^2 = c^2$

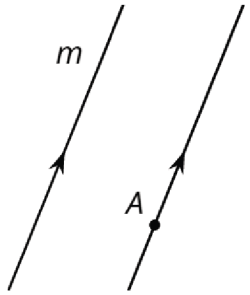
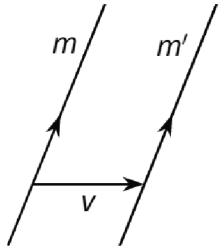
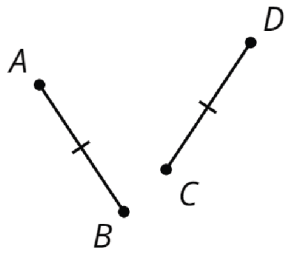
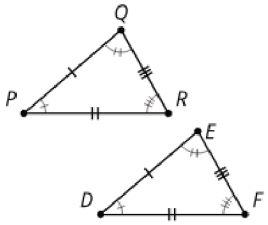
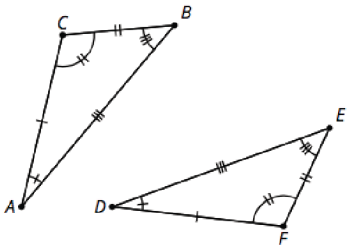
Date, Type	Statement	Diagram
U3, L6 definition	The **cosine** of an acute angle in a right triangle is the ratio (quotient) of the length of the adjacent leg to the length of the hypotenuse.	 $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$
U3, L6 definition	The **sine** of an acute angle in a right triangle is the ratio (quotient) of the length of the opposite leg to the length of the hypotenuse.	 $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$
U3, L6 definition	The **tangent** of an acute angle in a right triangle is the ratio (quotient) of the length of the opposite leg to the length of the adjacent leg.	 $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
U3, L10 definition	The **arccosine** of a number between 0 and 1 is the measure of an acute angle whose cosine is that number.	 $\arccos\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right) = \theta$
U3, L10 definition	The **arcsine** of a number between 0 and 1 is the measure of an acute angle whose sine is that number.	 $\arcsin\left(\frac{\text{opposite}}{\text{hypotenuse}}\right) = \theta$

Date, Type	Statement	Diagram
U3, L10 definition	The **arctangent** of a positive number is the measure of an acute angle whose tangent is that number.	 $\arctan\left(\frac{\text{opposite}}{\text{adjacent}}\right) = \theta$
U7, L6 assertion	**Inscribed Angle Theorem:** The measure of an inscribed angle is half the measure of the central angle that defines the same arc.	 $m\angle BCA = \frac{1}{2}m\angle BOA$
U7, L7 theorem	A line is **tangent** to a circle if and only if it is perpendicular to the radius drawn to the point of tangency.	 $\overline{AB} \perp \ell$
U7, L9 theorem	The three perpendicular bisectors of the sides of a triangle meet at a single point, called the triangle's **circumcenter**. This point is the center of the triangle's circumscribed circle.	
U7, L11 theorem	The three angle bisectors of a triangle meet at a single point, called the triangle's **incenter**. This point is the center of the triangle's inscribed circle.	

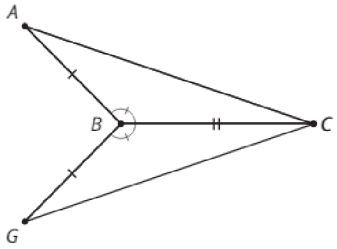
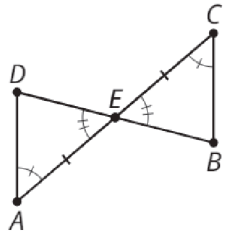
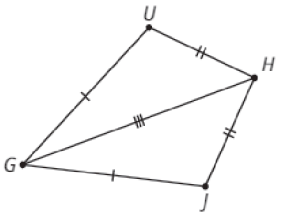
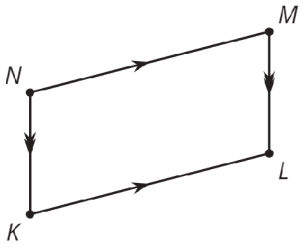
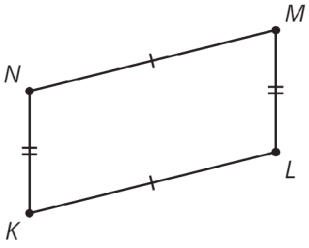
Date, Type	Statement	Diagram
U7, L12 theorem	To calculate the **area of a sector** or the **length of an arc**, first find the fraction of the circle represented by the central angle of the arc or sector. Multiply this fraction by the circle's area or circumference.	 arc length: 3π cm sector area: 6π cm^2
U7, L15 definition	For any angle, imagine drawing a circle with the angle's vertex at its center. Then, the "**radian** measure of the angle" is the ratio of the length of the arc defined by the angle to the circle's radius. That is, $\theta = \frac{arc\ length}{radius}$.	

type, number	statement	diagram
Assertion 1	A **rigid transformation** is a translation, reflection, rotation, or any sequence of the three. Rigid transformations take lines to lines, angles to angles of the same measure, and segments to segments of the same length.	
Definition 2	One figure is **congruent** to another if there is a sequence of translations, rotations, and reflections that takes the first figure exactly onto the second figure. The second figure is called the image of the rigid transformation.	 $\triangle EDC \cong \triangle E'D'C'$
Definition 3	**Reflection** is a rigid transformation that takes a point to another point that is the same distance from the given line, is on the other side of the given line, and so that the segment from the original point to the image is perpendicular to the given line. "Reflect _(object)_ across line _(name)_."	 Reflect *A* across line *m*.
Definition 4	**Translation** is a rigid transformation that takes a point to another point so that the directed line segment from the original point to the image is parallel to the given line segment and has the same length and direction. "Translate _(object)_ by the directed line segment _(name or from [point] to [point])_."	 Translate *A* by the directed line segment *v*.
Definition 5	**Rotation** is a rigid transformation that takes a point to another point on the circle through the original point with the given center. The two radii to the original point and the image make the given angle. "Rotate _(object)_ (clockwise or counterclockwise) by _(angle or angle measure)_ using center _(point)_."	 Rotate *P* counterclockwise by *a*° using center *C*.

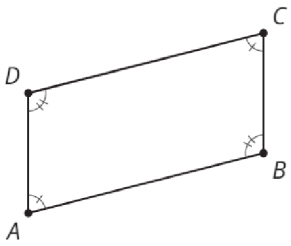
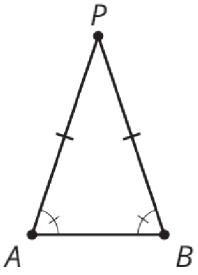
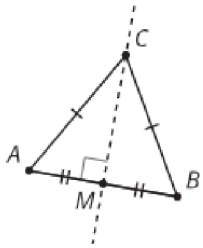
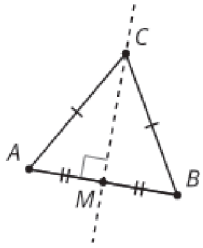
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type, number	statement	diagram
Assertion 6	**Parallel Postulate:** Given a line *m* and a point *A* that is not on *m*, there is exactly one line that goes through *A* that is parallel to *m*.	
Theorem 7	Translations take lines to parallel lines or to themselves.	 $m \parallel m'$
Theorem 8	If two segments have the same length, then they are congruent.	 $AB=CD$, so $\overline{AB} \cong \overline{CD}$
Theorem 9	If two figures are congruent, then **corresponding** parts of those figures must be congruent	 $\triangle PQR \cong \triangle DEF$ so $PQ=DE$, $PR=DF$, $QR=EF$, $\angle P \cong \angle D$, $\angle Q \cong \angle E$, $\angle R \cong \angle F$
Theorem 10	If all pairs of corresponding sides and all pairs of corresponding angles are congruent, then the triangles must be congruent.	 $AB=DE$, $BC=EF$, $CA=FD$, $\angle B \cong \angle E$, $\angle A \cong \angle D$, $\angle C \cong \angle F$ so $\triangle ABC \cong \triangle DEF$

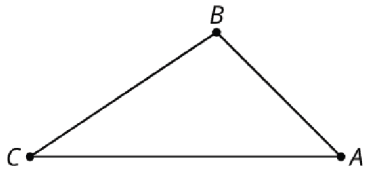
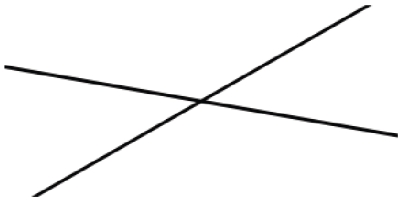
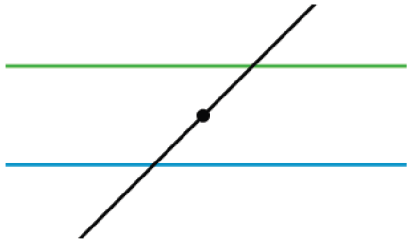
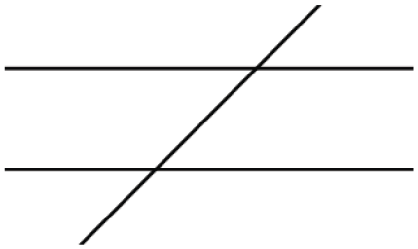
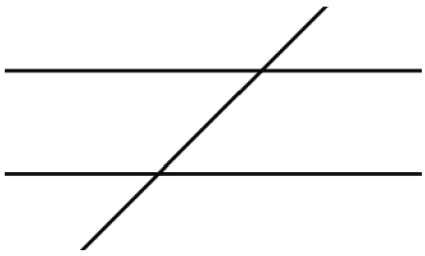
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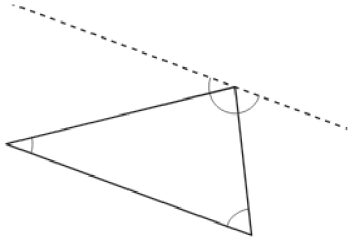
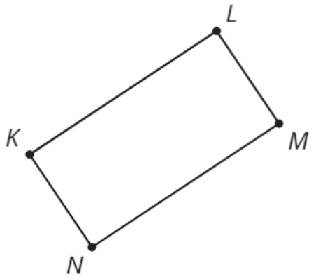
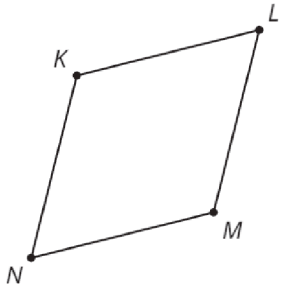
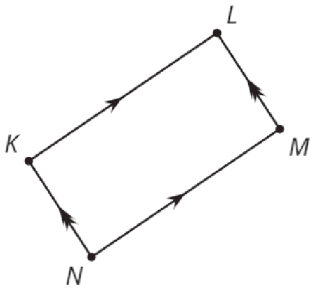
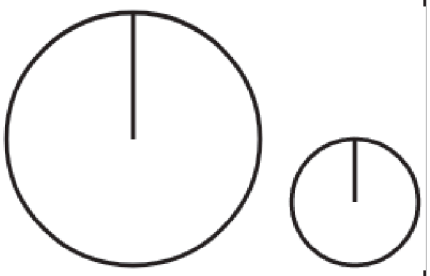
type, number	statement	diagram
Theorem 11	**Side-Angle-Side Triangle Congruence Theorem:** In two triangles, if two pairs of congruent corresponding sides and the pair of corresponding angles between the sides are congruent, then the two triangles are congruent.	 *AB=GB*, *BC=BC*, $\angle ABC \cong \angle GBC$ so $\triangle ABC \cong \triangle GBC$
Theorem 12	**Angle-Side-Angle Triangle Congruence Theorem:** In two triangles, if two pairs of corresponding angles, and the pair of corresponding sides between the angles, are congruent, then the triangles must be congruent.	 $\angle A \cong \angle C$, *AE=EC*, $\angle DEA \cong \angle BEC$, so $\triangle DEA \cong \triangle BEC$
Theorem 13	**Side-Side-Side Triangle Congruence Theorem:** In two triangles, if all three pairs of corresponding sides are congruent, then the triangles must be congruent.	 *HU=HJ*, *UG=JG*, *HG=HG* so $\triangle HUG \cong \triangle HJG$
Definition 14	A **parallelogram** is a quadrilateral with two pairs of opposite sides parallel.	 *NM* // *KL*, *NK* // *ML*, so *MNKL* is a parallelogram
Theorem 15	In a parallelogram, pairs of opposite sides are congruent.	 *MNKL* is a parallelogram, so *NM=KL*, *NK=ML*

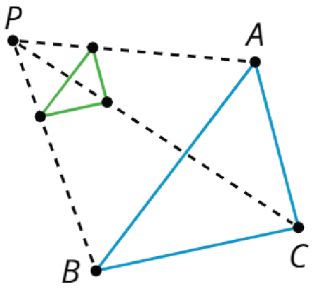
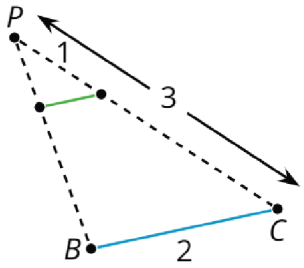
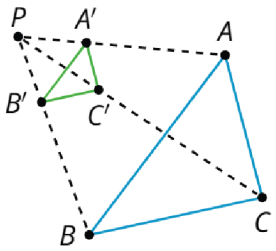
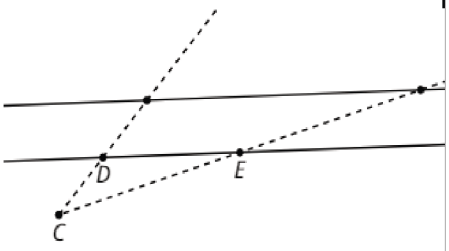
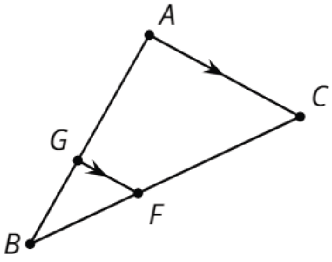
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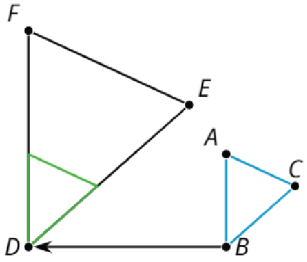
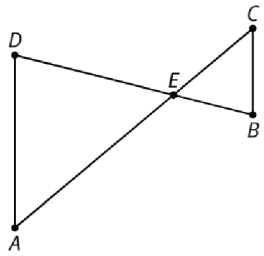
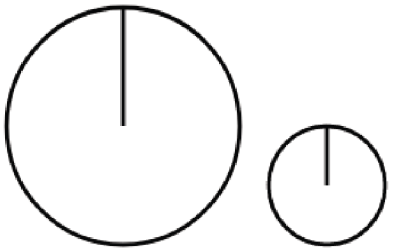
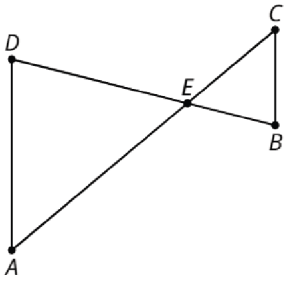
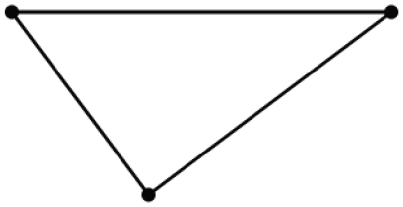
type, number	statement	diagram
Theorem 16	In a parallelogram, opposite angles are congruent.	 *ABCD* is a parallelogram, so $\angle A \cong \angle C$, $\angle D \cong \angle B$
Theorem 17	**Isosceles Triangle Theorem:** In an isosceles triangle, the base angles are congruent.	 *AP=PB* so $\angle A \cong \angle B$
Theorem 18	If a point *C* is the same distance from *A* as it is from *B*, then *C* must be on the perpendicular bisector of *AB*.	 *AC=BC*, *M* is the midpoint, so $MC \perp AB$
Theorem 19	If *C* is a point on the perpendicular bisector of segment *AB*, the distance from *C* to *A* is the same as the distance from *C* to *B*.	 $AB \perp CM$, *AM=BM*, so *AC=BC*

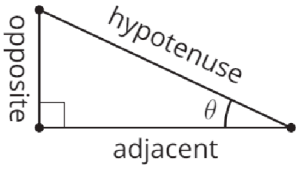
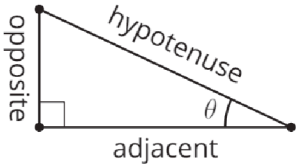
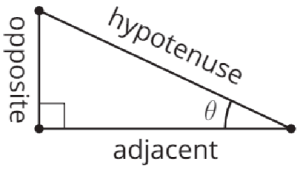
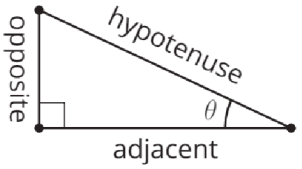
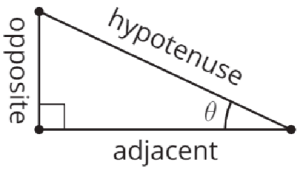
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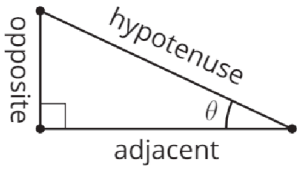
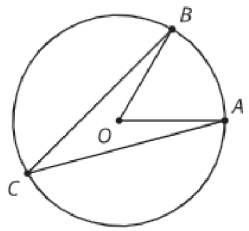
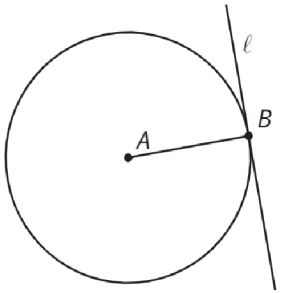
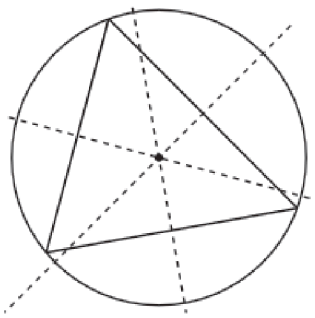
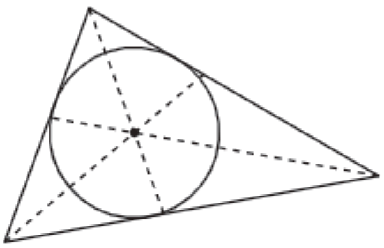
date, type	statement	diagram
theorem	If a triangle has side lengths _____, _____, and _____, then _____.	
theorem	_____ angles are _____.	
assertion	_____ by _____ takes lines to _____ lines or to _____.	
theorem	_____ **Angle Theorem:** If two _____ lines are cut by a _____, then alternate interior angles are _____. Conversely, if two lines are cut by a _____ and alternate interior angles are _____, then the lines have to be _____.	
theorem	_____ **Angle Theorem:** If two _____ lines are cut by a _____, then corresponding angles are _____. Conversely, if two _____ are cut by a _____ and corresponding angles are congruent, then the lines have to be _____.	

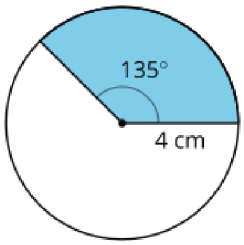
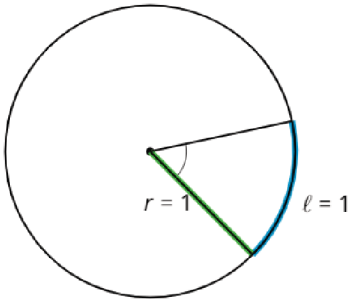
lesson, type	statement	diagram
theorem	**Triangle _____ Theorem:** The three _____ measures of any _____ always sum to _____ degrees.	
definition	A _____ is a quadrilateral with four _____.	
definition	A _____ is a quadrilateral with four _____ sides.	
theorem	If a _____ has (at least) one _____, then it is a _____.	
definition	_____ is the factor by which every _____ in an original figure is _____ when you make a _____ copy.	

date, type	statement	diagram
definition	A _____ with center *P* and positive _____ *k* takes a point *A* along the _____ *PA* to another point whose _____ is *k* times further away from *P* than _____ is. Dilate _(object)_ using center _(point)_ and a scale factor of _(number)_.	
assertion	The _____ of a line segment is _____ or shorter according to the same _____ given by the _____.	
assertion	If a figure is _____, then corresponding _____ are _____.	
theorem	A _____ takes a line not passing through the _____ of the dilation to a _____ line, and leaves a line passing through the _____ unchanged.	
theorem	If a line divides two _____ of a triangle proportionally, the _____ must be _____ to the _____ of the triangle.	

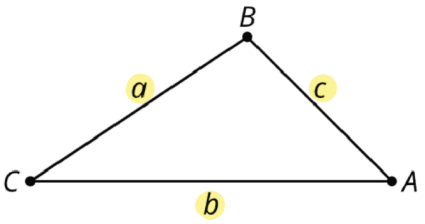
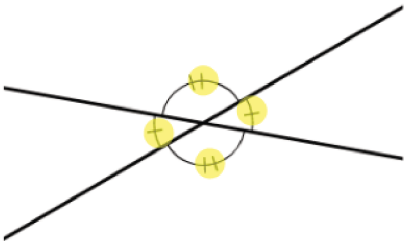
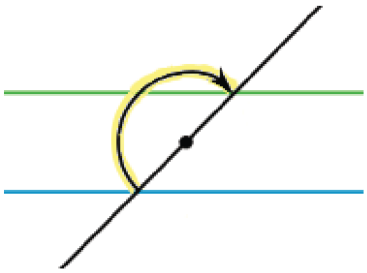
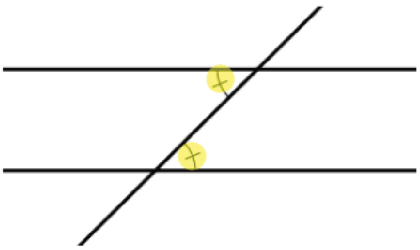
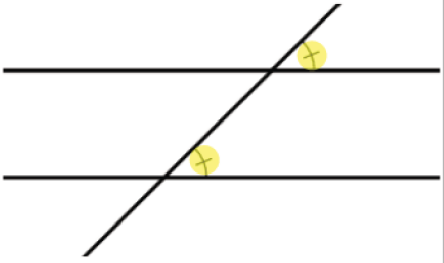
date, type	statement	diagram
definition	One figure is _____ to another if there is a sequence of _____ and _____ that takes the first figure so that it fits _____ over the second.	
theorem	If two _____ have all pairs of corresponding _____ congruent, and all pairs of corresponding _____ in the same proportion, then the two triangles are _____.	 $\angle A \cong \angle C$, $\angle D \cong \angle B$, $\angle DEA \cong \angle BEC$, $\frac{AD}{CB} = \frac{DE}{BE} = \frac{EA}{EC}$ so
theorem	All _____.	
theorem	_____ **Triangle Similarity Theorem:** In two _____, if _____ pairs of corresponding _____ are congruent, then the triangles must be _____.	
theorem	_____ **Theorem:** If a _____ triangle has _____ with lengths _____ and _____ and hypotenuse with length *c*, then _____.	

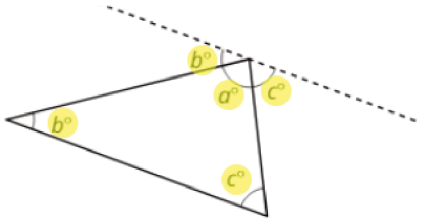
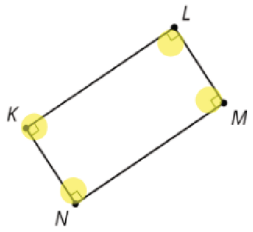
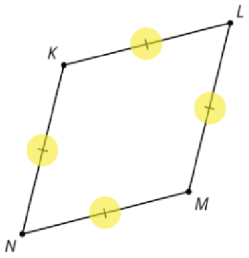
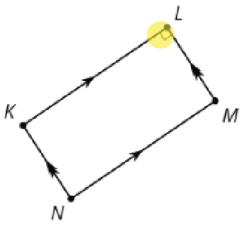
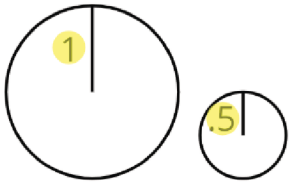
date, type	statement	diagram
definition	The _____ of an acute angle in a _____ triangle is the ratio (quotient) of the length of the _____ leg to the length of the _____.	
definition	The _____ of an acute angle in a _____ triangle is the ratio (quotient) of the length of the _____ leg to the length of the _____.	
definition	The _____ of an acute angle in a _____ triangle is the ratio (quotient) of the length of the _____ leg to the length of the _____ leg.	
definition	The _____ of a number between _____ and _____ is the measure of an acute _____ whose _____ is that number.	
definition	The _____ of a number between _____ and _____ is the measure of an acute _____ whose _____ is that number.	

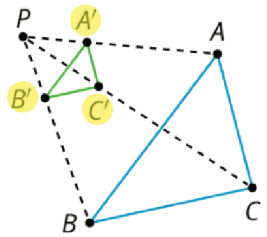
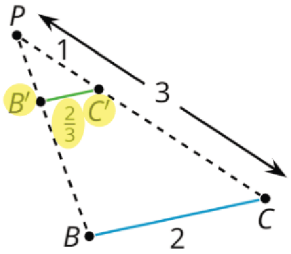
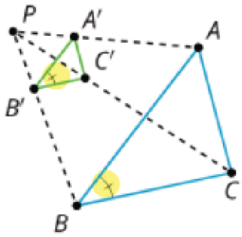
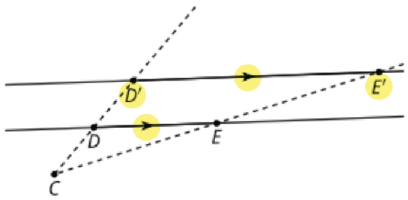
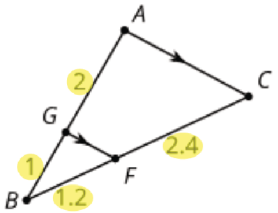
date, type	statement	diagram
definition	The _____ of a positive number is the measure of an acute _____ whose _____ is that number.	
assertion	_____ **Angle Theorem**: The measure of an _____ angle is _____ the measure of the _____ angle that defines the same arc.	
theorem	A _____ is _____ to a _____ if and only if it is _____ to the radius drawn to the point of _____.	
theorem	The three _____ of the sides of a triangle meet at a single _____, called the triangle's _____. This point is the _____ of the triangle's _____.	
theorem	The three _____ of a triangle meet at a single _____, called the triangle's _____. This point is the _____ of the triangle's _____.	

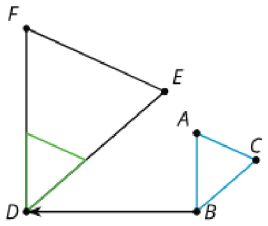
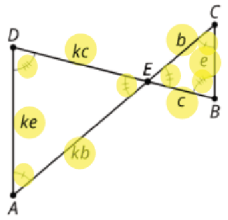
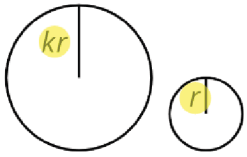
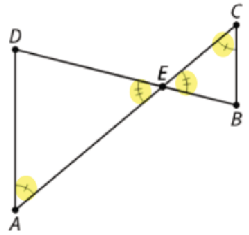
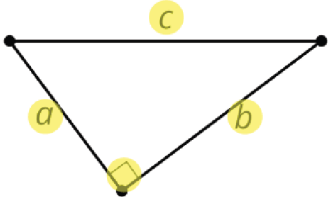
date, type	statement	diagram
theorem	To calculate the _____ of a _____ or the _____ of an _____, first find the fraction of the circle represented by the central angle of the arc or sector. Multiply this _____ by the circle's _____ or _____.	
definition	For any _____, imagine drawing a _____ with the angle's vertex at its _____. Then, the "_____ measure of the angle" is the ratio of the length of the arc defined by the angle to the circle's radius. That is, _____	

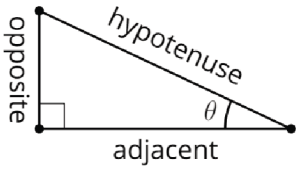
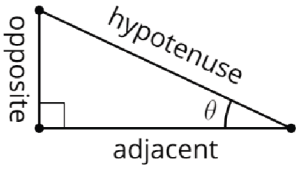
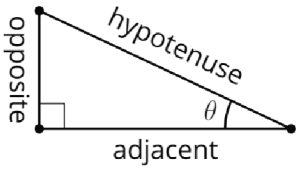
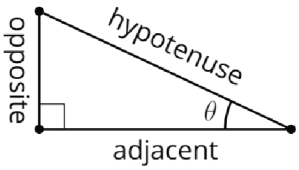
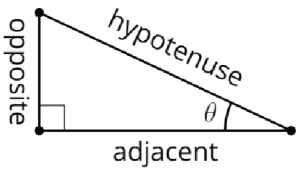
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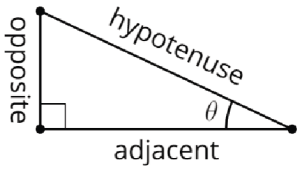
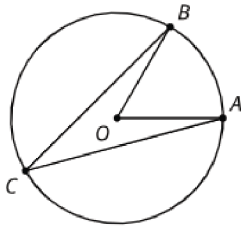
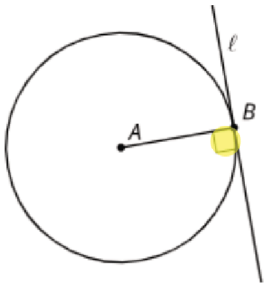
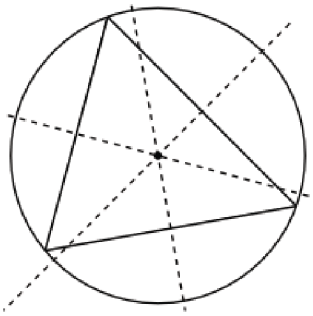
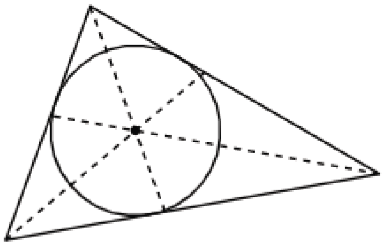
lesson, type	statement	diagram
U1, L3 (students write the date) theorem	If a triangle has side lengths a, b, and c, then $c < a + b$.	
U1, L7 theorem	Vertical angles are congruent.	
U1, L9 assertion	Rotation by 180 degrees takes lines to parallel lines or to themselves.	
U1, L9 theorem	**Alternate Interior Angle Theorem:** If two parallel lines are cut by a transversal, then alternate interior angles are congruent. Conversely, if two lines are cut by a transversal and alternate interior angles are congruent, then the lines have to be parallel.	
U1, L9 theorem	**Corresponding Angle Theorem:** If two parallel lines are cut by a transversal, then corresponding angles are congruent. Conversely, if two lines are cut by a transversal and corresponding angles are congruent, then the lines have to be parallel.	

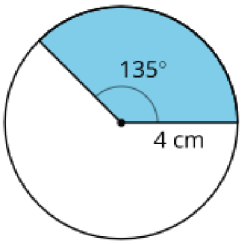
lesson, type	statement	diagram
U1, L10 theorem	**Triangle Angle Sum Theorem:** The three angle measures of any triangle always sum to 180 degrees.	 $a + b + c = 180$
U1, L12 definition	A **rectangle** is a quadrilateral with four right angles.	
U1, L12 definition	A **rhombus** is a quadrilateral with four congruent sides.	
U1, L12 theorem	If a parallelogram has (at least) one right angle, then it is a rectangle.	 *KLMN* has a right angle so it is a rectangle
U2, L1 definition	**Scale factor** is the factor by which every length in an original figure is multiplied when you make a scaled copy.	 Scale factor is 2 or ½

Date, Type	Statement	Diagram
U2, L1 definition	A **dilation** with center P and positive scale factor k takes a point A along the ray PA to another point whose distance is k times further away from P than A is. Dilate _(object)_ using center _(point)_ and a scale factor of _(number)_.	 $PA' = k \cdot PA$
U2, L3 assertion	The dilation of a line segment is longer or shorter according to the same ratio given by the scale factor.	 $PC:PC' = 3:1$, $BC:B'C' = 2:\frac{2}{3}$
U2, L4 assertion	If a figure is dilated, then corresponding angles are congruent.	 $\triangle A'B'C'$ is a dilation of $\triangle ABC$ so $\angle B \cong \angle B'$
U2, L4 theorem	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	 Dilate using center C. $DE \parallel D'E'$
U2, L5 theorem	If a line divides two sides of a triangle proportionally, the line must be parallel to the third side of the triangle.	 $\frac{1}{2} = \frac{1.2}{2.4}$ so $AC \parallel GF$

Date, Type	Statement	Diagram
U2, L6 definition	One figure is **similar** to another if there is a sequence of rigid motions and dilations that takes the first figure so that it fits exactly over the second.	 Translation and dilation takes $\triangle ABC$ onto $\triangle FDE$ so $\triangle ABC \sim \triangle FDE$
U2, L7 theorem	If two triangles have all pairs of corresponding angles congruent and all pairs of corresponding side lengths in the same proportion, then the two triangles are similar.	 .. $\angle A \cong \angle C$, $\angle D \cong \angle B$, $\angle DEA \cong \angle BEC$, $\frac{AD}{CB} = \frac{DE}{BE} = \frac{EA}{EC}$ so $\triangle DEA \sim \triangle BEC$
U2, L8 theorem	All circles are similar.	
U2, L9 theorem	**Angle-Angle Triangle Similarity Theorem:** In two triangles, if two pairs of corresponding angles are congruent, then the triangles must be similar.	 $\angle A \cong \angle C$, $\angle DEA \cong \angle BEC$, so $\triangle DEA \sim \triangle BEC$
U2, L16 theorem	**Pythagorean Theorem:** If a right triangle has legs with lengths a and b and hypotenuse with length c, then $a^2 + b^2 = c^2$.	 $a^2 + b^2 = c^2$

Date, Type	Statement	Diagram
U3, L6 definition	The **cosine** of an acute angle in a right triangle is the ratio (quotient) of the length of the adjacent leg to the length of the hypotenuse.	 $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$
U3, L6 definition	The **sine** of an acute angle in a right triangle is the ratio (quotient) of the length of the opposite leg to the length of the hypotenuse.	 $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$
U3, L6 definition	The **tangent** of an acute angle in a right triangle is the ratio (quotient) of the length of the opposite leg to the length of the adjacent leg.	 $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
U3, L10 definition	The **arccosine** of a number between 0 and 1 is the measure of an acute angle whose cosine is that number.	 $\arccos\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right) = \theta$
U3, L10 definition	The **arcsine** of a number between 0 and 1 is the measure of an acute angle whose sine is that number.	 $\arcsin\left(\frac{\text{opposite}}{\text{hypotenuse}}\right) = \theta$

Date, Type	Statement	Diagram
U3, L10 definition	The **arctangent** of a positive number is the measure of an acute angle whose tangent is that number.	 $\arctan\left(\frac{\text{opposite}}{\text{adjacent}}\right) = \theta$
U7, L6 assertion	**Inscribed Angle Theorem:** The measure of an inscribed angle is half the measure of the central angle that defines the same arc.	 $m\angle BCA = \frac{1}{2}m\angle BOA$
U7, L7 theorem	A line is tangent to a circle if and only if it is perpendicular to the radius drawn to the point of tangency.	 $\overline{AB} \perp \ell$
U7, L9 theorem	The three perpendicular bisectors of the sides of a triangle meet at a single point, called the triangle's **circumcenter**. This point is the center of the triangle's circumscribed circle.	
U7, L11 theorem	The three angle bisectors of a triangle meet at a single point, called the triangle's **incenter**. This point is the center of the triangle's inscribed circle.	

Date, Type	Statement	Diagram
U7, L12 theorem	To calculate the area of a sector or the length of an arc, first find the fraction of the circle represented by the central angle of the arc or sector. Multiply this fraction by the circle's area or circumference.	 arc length: 3π cm sector area: 6π cm^2
U7, L15 definition	For any angle, imagine drawing a circle with the angle's vertex at its center. Then, the "**radian** measure of the angle" is the ratio of the length of the arc defined by the angle to the circle's radius. That is, $\theta = \frac{arc\ length}{radius}$.	