

# A Towering Sequence

Let's explore the Tower of Hanoi.

## 1.1

## Which Three Go Together: What's Next?

Which three go together? Why do they go together?

A

2, 3, 5, 9, 17

B

2, 3, 4.5, 6.75

C

2, 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$

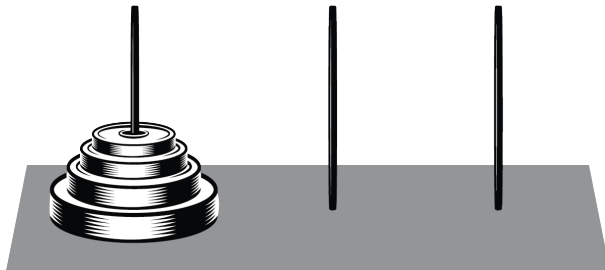
D

2, -4, 8, -16, 32

## 1.2

## The Tower of Hanoi

In the Tower of Hanoi (huh-NOY) puzzle, a set of discs sits on 1 peg, and there are 2 other empty pegs.



A *move* in the Tower of Hanoi puzzle involves taking a disc and moving it to another peg. There are two rules:

- Move only 1 disc at a time.
- Never put a larger disc on top of a smaller one.

You complete the puzzle by building the complete tower on any peg other than the starting peg.

1. Using 3 discs, complete the puzzle. What is the smallest number of moves you can find?
2. Using 4 discs, complete the puzzle. What is the smallest number of moves you can find?
3. Jada says she used the solution for 3 discs to help her solve the puzzle for 4 discs. Describe how this might happen.



### Are you ready for more?

What if a Tower of Hanoi puzzle with 64 discs was being solved at a rate of 1 move per second. How long would it take to solve this puzzle? Explain how you know.

## 1.3

## Checker Jumping Puzzle

Some checkers are lined up. Blue checkers are on one side, red are on the other, and there is 1 empty space between them. A *move* in this checker game pushes any checker forward 1 space or jumps over any 1 checker of the other color. Jumping the same color is not allowed, moving backward is not allowed, and 2 checkers cannot occupy the same space.



The puzzle is completed when the colors are completely switched: ending up with blue on the right, red on the left, and 1 empty space between them.

1. Using 1 checker on each side, complete the puzzle. What is the smallest number of moves needed?
2. Using 3 checkers on each side, complete the puzzle. What is the smallest number of moves needed?
3. Estimate the number of moves needed if there are 4 checkers on each side, then test your guess.
4. Noah says he used the solution for 3 checkers on each side to help him solve the puzzle for 4 checkers. Describe how this might happen.

## Lesson 1 Summary

A list of numbers, like 3, 5, 7, 9, 11, ... or 1, 5, 13, 29, 61, ..., is called a **sequence**.

There are many ways to define a sequence, but one way is to describe how each **term** relates to the term before it. For example, the sequence 3, 5, 7, 9, 11, ... can be described this way: the starting term is 3, then each following term is 2 more than the term before it. The sequence 1, 5, 13, 29, 61, ... can be described like this: the starting term is 1, then each following term is the sum of 3 and twice the previous term.

Throughout this unit, we will study several types of sequences along with ways to represent them.