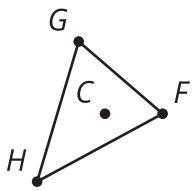
Unit 3, Lesson 3

# Measuring Dilations

Let's dilate polygons.

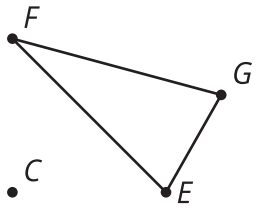
## 3.1 Dilating Out

Dilate triangle $FGH$ using center $C$ and a scale factor of 3.



## 3.2 All the Scale Factors

Here is a center of dilation and a triangle.



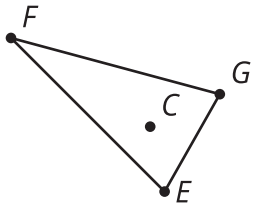

1. Measure the sides of triangle $EFG$ (to the nearest mm).

2. Your teacher will assign you a scale factor. Predict the relative lengths of the original figure and the image after you dilate by your scale factor.

3. Dilate triangle $EFG$ using center $C$ and your scale factor.
4. How does your prediction compare to the image you drew?

5. Use tracing paper to copy point $C$, triangle $EFG$, and your dilation. Label your tracing paper with your scale factor.
6. Align your tracing paper with your partner's. What do you notice?

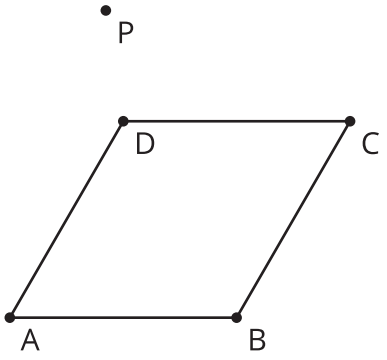
## Are you ready for more?




1. Dilate triangle $EFG$ using center $C$ and scale factors:
   a. $\frac{1}{2}$
   b. 2
2. What scale factors would cause some part of triangle $E'F'G'$ to intersect some part of triangle $EFG$?

## 3.3 What Stays the Same?

1. Dilate quadrilateral $ABCD$ using center $P$ and your scale factor.




2. Complete the table.

| Ratio | $\frac{PA'}{PA}$ | $\frac{PB'}{PB}$ | $\frac{PC'}{PC}$ | $\frac{PD'}{PD}$ |
|---|---|---|---|---|
| Value | | | | |

3. What do you notice? Can you prove your conjecture?

4. Complete the table.

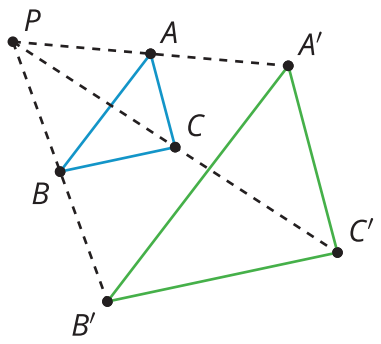
| Ratio | $\frac{B'A'}{BA}$ | $\frac{C'B'}{CB}$ | $\frac{D'C'}{DC}$ | $\frac{A'D'}{AD}$ |
|---|---|---|---|---|
| Value | | | | |

5. What do you notice? Does the same reasoning you just used also prove this conjecture?

## Lesson 3 Summary

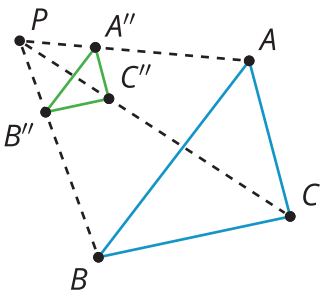
We know that a *dilation* with center $P$ and positive *scale factor*, $k$, takes a point $A$ along the ray $PA$ to another point whose distance is $k$ times farther away from $P$ than $A$ is.

The triangle $A'B'C'$ is a dilation of the triangle $ABC$ with center $P$ and with a scale factor of 2. So $A'$ is 2 times farther away from $P$ than $A$ is, $B'$ is 2 times farther away from $P$ than $B$ is, and $C'$ is 2 times farther away from $P$ than $C$ is.




Because of the way dilations are defined, all of these quotients give the scale factor:
$\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = 2.$

If triangle $ABC$ is dilated from point $P$ with scale factor $\frac{1}{3}$, then each vertex in $A''B''C''$ is on the ray from P through the corresponding vertex of $ABC$, and the distance from $P$ to each vertex in $A''B''C''$ is one-third as far as the distance from $P$ to the corresponding vertex in $ABC$.




$\frac{PA''}{PA} = \frac{PB''}{PB} = \frac{PC''}{PC} = \frac{1}{3}$

The dilation of a line segment is longer or shorter according to the same ratio given by the scale factor. In other words, if segment $AB$ is dilated from point $P$ with a scale factor of $k$, then the length of segment $AB$ is multiplied by $k$ to get the corresponding length of $A'B'$.

$\frac{A''B''}{AB} = \frac{B''C''}{BC} = \frac{A''C''}{AC} = k.$

Corresponding side lengths of the original figure and dilated image are all in the same proportion, and are related by the same scale factor, $k$.