Unit 4, Lesson 2

Solutions to Inequalities in One Variable

Let's find and interpret solutions to inequalities in one variable.

2.1 Find a Value, Any Value

1. Write 3 solutions to the inequality $y \leq 9.2$. Be prepared to explain what makes a value a solution to this inequality.

2. Write one solution to the inequality $7(3 - x) > 14$. Be prepared to explain your reasoning.

2.2 Off to an Orchard

A teacher is choosing between two options for a class field trip to an orchard.

- At Orchard A, admission costs $9 per person and 3 chaperones are required.
- At Orchard B, the cost is $10 per person, but only 1 chaperone is required.
- At each orchard, the same price applies to both chaperones and students.

1. Which orchard would be cheaper to visit if the class has:
   a. 8 students?
   b. 12 students?
   c. 30 students?

2. To help her compare the cost of her two options, the teacher first writes the equation $9(n+3) = 10(n+1)$, and then she writes the inequality $9(n+3) < 10(n+1)$.

   a. What does n represent in each statement?

   b. In this situation, what does the equation $9(n+3) = 10(n+1)$ mean?

   c. What does the solution to the inequality $9(n+3) < 10(n+1)$ tell us?

   d. Graph the solution to the inequality on the number line. Be prepared to show or explain your reasoning.

2.3 Part-Time Work

To help pay for his tuition, a college student plans to work in the evenings and on weekends. He has been offered two part-time jobs: working in the guest-services department at a hotel and waiting tables at a popular restaurant.

- The job at the hotel pays $18 an hour and offers $33 in transportation allowance per month.
- The job at the restaurant pays $7.50 an hour plus tips. The entire wait staff typically collects about $50 in tips each hour. Tips are divided equally among the 4 wait-staff members who share a shift.

1. The equation $7.50h + \frac{50}{4}h = 18h + 33$ represents a possible constraint about the situation.

   a. Solve the equation and check your solution.

b. Here is a graph on a number line.

Put a scale on the number line so that the point marked with a circle represents the solution to the equation.

2. Does one job pay better if:

   a. The student works fewer hours than the solution you found earlier? If so, which job?

   b. The student works more hours than the solution you found earlier? If so, which job?

Be prepared to explain or show how you know.

3. Here are two inequalities and two graphs that represent the solutions to the inequalities.

   - Inequality 1: $7.50h + \frac{50}{4}h < 18h + 33$
   - Inequality 2: $7.50h + \frac{50}{4}h > 18h + 33$

A

B

   a. Put the same scale on each number line so that the circle represents the number of hours that you found earlier.

   b. Match each inequality with a graph that shows its solution. Be prepared to explain or show how you know.

2.4 Equality and Inequality

1. Solve this equation, and check your solution: $\frac{4(x+3)}{5} = 4x - 12$.

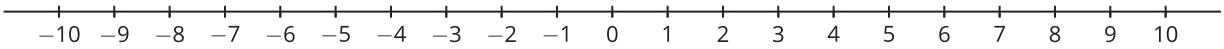
2. Consider the inequality: $\frac{4(x+3)}{5} \leq 4x - 12$.

   a. Choose a couple of values less than 2 for x. Are they solutions to the inequality?

   b. Choose a couple of values greater than 2 for x. Are they solutions to the inequality?

   c. Choose 2 for x. Is it a solution?

   d. Graph the solution to the inequality on the number line.



Are you ready for more?

Here is a different type of inequality: $x^2 \leq 4$.

1. Is 1 a solution to the inequality? Is 3 a solution? How about -3?
2. Describe all solutions to this inequality. (If you like, you can graph the solutions on a number line.)
3. Describe all solutions to the inequality $x^2 \geq 9$. Test several numbers to make sure that your answer is correct.

2.5 More or Less?

Consider the inequality $-\frac{1}{2}x + 6 < 4x - 3$. Let's look at another way to find its solutions.

1. Use graphing technology to graph $y = -\frac{1}{2}x + 6$ and $y = 4x - 3$ on the same coordinate plane.
2. Use your graphs to answer the following questions:
   a. Find the values of $-\frac{1}{2}x + 6$ and $4x - 3$ when x is 1.

   b. What value of x makes $-\frac{1}{2}x + 6$ and $4x - 3$ equal?

   c. For what values of x is $-\frac{1}{2}x + 6$ less than $4x - 3$?

   d. For what values of x is $-\frac{1}{2}x + 6$ greater than $4x - 3$?

3. What is the solution to the inequality $-\frac{1}{2}x + 6 < 4x - 3$? Be prepared to explain how you know.

Lesson 2 Summary

The equation $\frac{1}{2}t = 10$ is an equation in one variable. Its solution is any value of t that makes the equation true. Only $t = 20$ meets that requirement, so 20 is the only solution.

The inequality $\frac{1}{2}t > 10$ is an inequality in one variable. Any value of t that makes the inequality true is a solution. For instance, 30 and 48 are both solutions because substituting these values for t produces true inequalities. $\frac{1}{2}(30) > 10$ is true, as is $\frac{1}{2}(48) > 10$. Because the inequality has a range of values that make it true, we sometimes refer to *all* the solutions as the *solution set*.

One way to find the solutions to an inequality is by reasoning. For example, to find the solution to $2p < 8$, we can reason that if 2 times a value is less than 8, then that value must be less than 4. So a solution to $2p < 8$ is any value of p that is less than 4.

Another way to find the solutions to $2p < 8$ is to solve the related equation $2p = 8$. In this case, dividing each side of the equation by 2 gives $p = 4$. This point, where p is 4, is the *boundary* of the solution to the inequality.

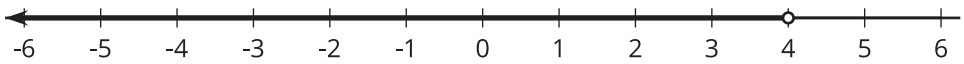
To find out the range of values that make the inequality true, we can try values less than and greater than 4 in our inequality and see which ones make a true statement.

Let's try some values less than 4:

- If $p = 3$, the inequality is $2(3) < 8$ or $6 < 8$, which is true.
- If $p = -1$, the inequality is $2(-1) < 8$ or $-2 < 8$, which is also true.

Let's try values greater than 4:

- If $p = 5$, the inequality is $2(5) < 8$ or $10 < 8$, which is false.
- If $p = 12$, the inequality is $2(12) < 8$ or $24 < 8$, which is also false.

In general, the inequality is false when p is greater than or equal to 4 and true when p is less than 4.

We can represent the solution set to an inequality by writing an inequality, $p < 4$, or by graphing on a number line. The ray pointing to the left represents all values less than 4.