Unit 4, Lesson 14

Solving Systems by Elimination (Part 1)

Goals

- Recognize that adding or subtracting equations in a system creates a new equation with a solution that coincides with that of the original system, so the new equation can be used to solve the original system.
- Solve systems of equations by adding or subtracting the equations strategically to eliminate a variable.
- Use graphing technology to graph the sums and differences of the equations in a system, and analyze and describe (orally and in writing) the behaviors of the graphs.

Learning Targets

- I can solve systems of equations by adding or subtracting them to eliminate a variable.
- I know that adding or subtracting equations in a system creates a new equation, where one of the solutions to this equation is the solution to the system.

Lesson Narrative

This is the first of three lessons that develop the idea of solving systems of linear equations in two variables by **elimination**.

Students examine a diagram of three hangers where the third hanger contains the combined contents of the first two hangers and all three hangers are balanced. Then, they analyze the result of adding two linear equations in standard form and notice that doing so eliminates one of the variables, enabling them to solve for the other variable and, consequently, to solve the system. In studying and testing a new strategy of adding equations and then offering their analyses, students construct viable arguments and critique the reasoning of others (MP3).

Next, students connect the solution they found using this method to the graphs of the equations in a system and the graph of the third equation (that results from adding or subtracting the original equations). They observe that the solution they found is the solution to the system, and that the graph of the third equation intersects the other two graphs at the exact same point—at the intersection of the first two.

The foundational idea is that adding or subtracting equations in a system creates a new equation whose solutions coincide with those of the original system. Students begin using this insight to solve systems, but they are not yet expected to construct an argument as to why this approach works.

Standards

Addressing	**HSA-REI.C.6, HSF-BF.A.1.b**
Building Toward	**HSA-REI.C.5**

Instructional Routines

- Graph It
- MLR1: Stronger and Clearer Each Time
- Notice and Wonder

Required Materials

Materials to Gather

- Graphing technology: Activity 3

Required Preparation

Activity 3:

Acquire devices that can run Desmos (recommended) or other graphing technology. It is ideal if students each have their own device. (If students typically access the digital version of the materials, Desmos is always available under Math Tools.)

Student Facing Learning Goals

Let’s investigate how adding or subtracting equations can help us solve systems of linear equations.

14.1 Notice and Wonder: Hanger Diagrams

Warm-up

5 min

Activity Narrative

The purpose of this *Warm-up* is to give students an intuitive and concrete way to think about combining two equations that are each true.

Students are presented with diagrams of three balanced hangers, which suggest that the weights on the two sides of each hanger are equal. Each side of the last hanger shows the combined objects from the corresponding side of the first two hangers. Students can reason that if 2 circles weigh the same as 1 square, and 1 circle and 1 triangle weigh the same as 1 pentagon, then the combined weight of 3 circles and 1 triangle should also be equal to the combined weight of 1 square and 1 pentagon.

Standards

Building Toward **HSA-REI.C.5**

Instructional Routines

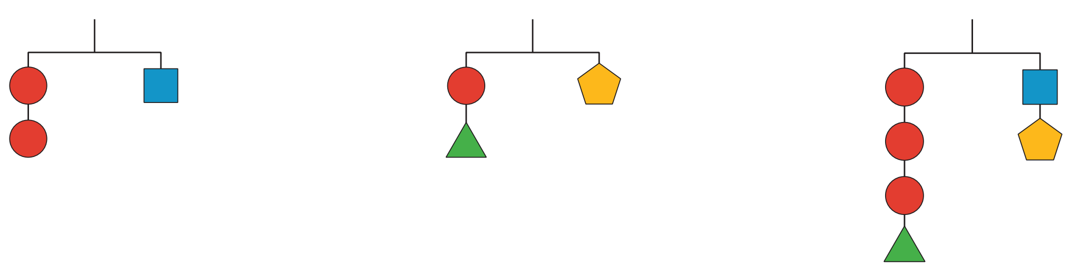
- Notice and Wonder

Launch

Arrange students in groups of 2. Display the hanger diagrams for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder about. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- All the hangers are balanced.
- There are circles on the left side of every hanger and none on the right side of any hanger.
- There is a triangle on the left side of the second and third hangers.
- There is a square on the right side of the first and third hangers.
- The third hanger has all the shapes from the first two hangers.
- The shapes from the left side of the first two hangers end up on the left side of the last hanger. (The same can be said about the right side.)

Things students may wonder:

- How much does each shape weigh?
- Is the circle the lightest shape?
- Which shape weighs the most?
- Why does the last hanger have many more shapes than the other two hangers?
- Why is the last hanger still balanced if it has shapes added to each side?

Activity Synthesis

Ask students to share the things that they noticed and wondered. Record and display, for all to see, their responses without editing or commentary. If possible, record the relevant reasoning on or near the diagrams. Next, ask students, "Is there anything on this list that you are wondering about now?" Encourage students to observe what is on display and to respectfully ask for clarification, point out contradicting information, or voice any disagreement.

The idea to emphasize is that the weights on each side of the third hanger come from combining the weights on the corresponding sides of the first two hangers. If no one points this out, raise it as a point for discussion. Ask students:

- "What do you notice about the left side of the last hanger? What about the right side?"
- "If we saw only the first two hangers but knew that the third hanger has the combined weights from the corresponding side of the first two hangers, could we predict whether the weights on the third hanger would balance? "Why or why not?" (Yes. We can think of it as adding the weights from the second hanger to the first one, or vice versa. If the same weight is added to each side of a balanced hanger, the hanger would still be balanced.)

14.2 Adding Equations

15 min

Activity Narrative

In the *Warm-up* of this lesson, students saw a visual representation of two equations being added to form a third equation. Because the first two equations are balanced, the third is also balanced. In this activity, students continue to develop the idea of adding two equations to form a third equation and use it to help them solve systems of linear equations.

Along the way, students examine the work of others and practice explaining their reasoning and critiquing that of others (MP3). They also see that sometimes adding equations is a productive way to solve systems, but other times it isn't.

Standards

Addressing HSA-REI.C.6, HSF-BF.A.1.b

Building Toward HSA-REI.C.5

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet time to think about the first set of questions and then time to share their thinking with their partner. Follow with a whole-class discussion before students proceed to the second set of questions.

Invite students to share their analysis of Diego's work—what Diego has done to solve the system and why he might have done it that way. Discuss questions such as:

- "In this case, what happens when the equations are added? Why might it be helpful to do so?" (The expressions with x add up to 0, so x is removed from the equation, making it possible to solve for y.)
- "How does finding the value of y help with solving the system?" (Once we know the value of one variable, we can use it to find the value of the other, by substituting it back into one of the equations and solving that equation.)
- "How can we be sure that $x = 1$ and $y = 2$ simultaneously make both equations true and is a solution to the system?" (We can substitute those values into the equations and see if the equations are true. We can also graph the system and see if it intersects at $(1, 2)$.)

Next, ask students to complete the remainder of the activity.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin by asking, "Does this problem remind anyone of something we have seen before?" If it is not mentioned, explicitly draw a connection between how the hanger diagrams are related in the Warm-up to how the equations are related in Diego's work. In both, the left and right sides are still equivalent when they are added together.. Connect the intuitive understanding of balance to familiar mathematical statements that do not require solving. Present strategically in the same format as Diego’s work. For instance, ask students if $2 + 2 = 4$ is a true and balanced statement. Then ask them if $3 + 1 = 4$ is also a true and balanced statement. When students have labeled these as true, add them together in the same arrangement as the upcoming problems, and ask students if the result ($5 + 3 = 8$), will therefore also be true and balanced. Then introduce Diego’s work.
Supports accessibility for: Memory, Attention

Student Task Statement

Diego is solving this system of equations:

$$\begin{cases} 4x + 3y = 10 \\ \text{-}4x + 5y = 6 \end{cases}$$

Here is his work:

$$\begin{array}{r} 4x + 3y = 10 \\ \text{-}4x + 5y = 6 \quad + \\ \hline 0 + 8y = 16 \\ y = 2 \end{array}$$

$$\begin{aligned} 4x + 3(2) &= 10 \\ 4x + 6 &= 10 \\ 4x &= 4 \\ x &= 1 \end{aligned}$$

1. Make sense of Diego’s work and discuss with a partner:

   a. What did Diego do to solve the system?

b. Is the pair of x and y values that Diego found actually a solution to the system? How do you know?

2. Does Diego's method work for solving these systems? Be prepared to explain or show your reasoning.

a. $\begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases}$

b. $\begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$

Student Response

1. Sample response:
   a. Diego added the two equations, and the sum is an equation with only one variable, y, which can be solved. He then substituted the y value into the first equation and solved for x.
   b. Yes, when the values are substituted back into the equations in the system, they make the equations true.
2. a. Yes. $x = 5$, $y = -6$. Sample reasoning: Adding the two equations give $3x = 15$ or $x = 5$. Substituting 5 for x in either one of the original equations gives $y = -6$.
   b. Sample responses:
      - No, adding the two equations doesn't get us anywhere. The sum is $16x + 12y = 44$, which doesn't make it easier to solve.
      - It works if the second equation is subtracted from the first instead of added to the first. The difference is $10y = 30$ or $y = 3$. If we substitute 3 for y in one of the original equations, we get $x = \frac{1}{2}$.

Activity Synthesis

Invite students to share their responses to the last set of questions, and discuss whether Diego's method works for solving the two systems. Ask students:

- "Why does adding the equations work for solving the system with $2x + y = 4$ and $x - y = 11$, but doesn't work for $8x + 11y = 37$ and $8x + y = 7$?" (In the latter system, the resulting equation still has two variables whose values we don't know.)
- "What if we subtract the equations? Would that help us solve the last system?" (Yes, subtracting the second equation from the first gives $10y = 30$ or $y = 3$, which we can then use to find the x-value.)
- "How is adding the two equations here like and unlike combining the shapes in the two hanger diagrams earlier? How is it different?" (It is alike in that the result is another equation with the combined parts on the left side staying on the left and the combined parts on the right staying on the right. It is unlike the hanger diagrams in that these equations use numbers and variables, and in one of the systems, some of the parts on the left added up to 0 and are not shown in the third equation.)

Make sure students see that if we choose to add or subtract strategically, in each of the new equations, one variable is eliminated, making it possible to solve for the other variable. When the value of that variable is substituted to either of the original equations, we can solve for the variable that was eliminated. Tell students that this method of solving a system is called solving by **elimination**.

Point out that there is nothing wrong about adding the equations in the last system. It simply doesn't get us anywhere closer to the solution and is therefore unproductive.

Access for English Language Learners

MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the final question. Invite listeners to ask questions and give feedback that will help clarify and strengthen their partner's ideas and writing. Give students

3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

14.3 Adding and Subtracting Equations to Solve Systems

15 min

Activity Narrative

Earlier, students saw that adding or subtracting the equations in a system creates a third equation that can help us solve the system. In this activity, students reconnect the idea of the solution to a system to the intersection of the graphs of the equations. They graph each original pair of equations and the equation that results from adding or subtracting them. They then observe that the graph of the third equation intersects the other two graphs at the exact same point—at the intersection of the first two.

At this point, students simply get a graphical confirmation that adding or subtracting equations can help them find the solution to a system. They are not yet expected to be able to articulate why this is the case. That understanding will be developed over a few upcoming lessons.

Standards

Addressing	**HSA-REI.C.6, HSF-BF.A.1.b**
Building Toward	**HSA-REI.C.5**

Instructional Routines

- Graph It

Launch

Remind students that earlier they added or subtracted pairs of equations to form new equations. Explain that they will now graph each pair of equations in the systems given earlier, as well as the third equation that came from adding or subtracting those equations, and then make some observations about them.

Arrange students into groups of 3, and provide access to graphing technology. Assign one system for each group member to graph. Ask students to discuss their observations after graphing.

If possible, make available graphing technology that allows users to enter linear equations in standard form, such as Desmos (available under Math Tools). Otherwise, give students time to rearrange the equations into a form that can be used with the technology and to check their equivalent equations. If time is limited, provide these equivalent equations:

System A

$$\begin{cases} y = -\frac{4}{3}x + \frac{10}{3} \\ y = \frac{4}{5}x + \frac{6}{5} \end{cases}$$

System B

$$\begin{cases} y = -2x + 4 \\ y = x - 11 \end{cases}$$

System C

$$\begin{cases} y = -\frac{8}{11}x + \frac{37}{11} \\ y = -8x + 7 \end{cases}$$

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for finding the sum or difference of the equations before they begin. Students can speak quietly to themselves, or share with a partner. If students struggle, direct students to use parentheses around the entirety of the equation being added or subtracted to enhance clarity and reduce errors.
Supports accessibility for: Memory, Organization

Student Task Statement

Here are three systems of equations that you saw earlier.

System A

$$\begin{cases} 4x + 3y = 10 \\ -4x + 5y = 6 \end{cases}$$

System B

$$\begin{cases} 2x + y = 4 \\ x - y = 11 \end{cases}$$

System C

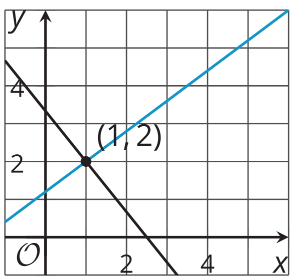
$$\begin{cases} 8x + 11y = 37 \\ 8x + y = 7 \end{cases}$$

1. For each system:
   a. Use graphing technology to graph the original two equations in the system. Then, identify the coordinates of the solution.
   b. Find the sum or difference of the two original equations that would enable the system to be solved.
2. What do you notice about the graph of the new third equation for each system? Make a conjecture about why the graph of the sum or difference is related in this way to the graph of the original equations.

Student Response

1.

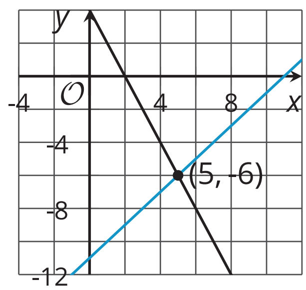
System A:



a.

b. Sample response: $8y = 16$ or (if the equations are subtracted) $y = 2$, or $8x - 2y = 4$.

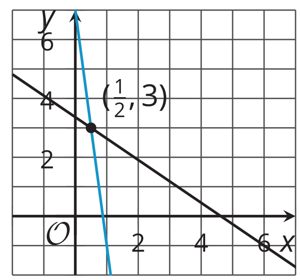
System B:



a.

b. Sample response: $3x = 15$ or $x = 5$.

System C:



a.

b. Sample response: $10y = 30$ or $-10y = -30$ or $y = 3$, or (if the equations are added) $16x + 12y = 46$.

2. Sample response: The graph of the new equation intersects the other two graphs at the same point. If the new equation is chosen to be most helpful, it will be a vertical or horizontal line. My conjecture is that the graph of the sum (or difference) will always intersect the graphs of the original equations in the same point because that point is the solution to the system and the sum (or difference) is an equivalent equation in the system.

Building on Student Thinking

When solving System B, some students may not notice that the y-variable in one equation has a positive coefficient and the other has a negative coefficient, and consequently decide to subtract the second equation from the first, rather than to add the two equations. They may struggle to figure out why the solution pair they find doesn't match what is on the graph. Suggest that they express the second equation in terms of addition, $x + (-y) = 11$, and try eliminating one variable again.

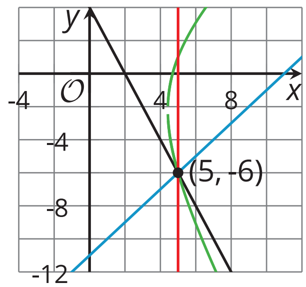
Are You Ready for More?

Mai wonders what would happen if we multiply equations. That is, we multiply the expressions on the left side of the two equations and set them equal to the product of the expressions on the right side of the two equations.

1. In system B write out an equation that you would get if you multiply the two equations in this manner.
2. Does your original solution still work in this new equation?
3. Use graphing technology to graph this new equation on the same coordinate plane. Why is this approach not particularly helpful?

Extension Student Response

1. $(2x + y)(x - y) = 44$ (or equivalent)
2. Yes.
3. Sample response: The new equation is not a line and therefore not so easy to work with.



Activity Synthesis

Display, for all to see, the graphs that students generated, and ask students to share their observations. Highlight that the graph of the new equation intersects the graphs of the equations in the original system at the same point.

Time permitting, ask students to subtract the equations that they previously added (or to add the equations that they previously subtracted) and then to graph the resulting equation on the same coordinate plane. Ask them to comment on the graphs. Students are likely to see that the graphs of the new equations are no longer horizontal or vertical lines, but they still intersect at the same point as the original graphs.

Invite students to share their conjectures as to why the graph of the new equation intersects the other two graphs at the same point. Without confirming or correcting their conjectures, tell students that they will investigate this question in the coming activities.

Lesson Synthesis

Now that students have, in their toolkit, an additional strategy for solving systems, invite students to reflect on three systems seen in the *Lesson Synthesis* of a previous lesson, in which they made a case for solving one system by substitution.

System 1

$$\begin{cases} 3m + n = 71 \\ 2m - n = 30 \end{cases}$$

System 2

$$\begin{cases} 4x + y = 1 \\ y = -2x + 9 \end{cases}$$

System 3

$$\begin{cases} 5x + 4y = 15 \\ 5x + 11y = 22 \end{cases}$$

Ask students to discuss the following questions with a partner and to be prepared to report their partner's responses:

- "Look at a system that you would have chosen to solve by substitution. Would you still choose to solve it by substitution now? Why or why not?"

- "Look at a system that you would *not* have chosen to solve by substitution. Would it help to solve by elimination? Why or why not?"

With a little bit of rearranging (of the equations), all of these systems could be solved by substitution or elimination. Students should at least recognize that systems 1 and 3 can be efficiently solved by elimination, while system 2 can be efficiently solved by substitution.

14.4 What to Do with This System?

Cool-down

5 min

Graphing technology should not be used in this *Cool-down*.

Standards

Addressing **HSA-REI.C.6**

Student Task Statement

Here is a system of linear equations: $\begin{cases} 2x + \frac{1}{2}y = 7 \\ 6x - \frac{1}{2}y = 5 \end{cases}$

1. Which would be more helpful for solving the system: adding the two equations or subtracting one from the other? Explain your reasoning.
2. Solve the system without graphing. Show your reasoning.

Student Response

1. Adding the equations. Sample explanation: $\frac{1}{2}y + \text{-}\frac{1}{2}y = 0$, so adding the equations would eliminate the y-variable and enable solving for x.
2. $x = \frac{3}{2}$ (or equivalent) and $y = 8$. Sample reasoning:

$$\begin{aligned} 2x + \tfrac{1}{2}y &= 7 \\ 6x - \tfrac{1}{2}y &= 5 \quad + \\ \hline 8x + \quad 0 &= 12 \\ 8x &= 12 \\ x &= \tfrac{3}{2} \end{aligned}$$

$$\begin{aligned} 2x + \tfrac{1}{2}y &= 7 \\ 2\left(\tfrac{3}{2}\right) + \tfrac{1}{2}y &= 7 \\ 3 + \tfrac{1}{2}y &= 7 \\ \tfrac{1}{2}y &= 4 \\ y &= 8 \end{aligned}$$

Responding to Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 14 Summary

Another way to solve systems of equations algebraically is by **elimination**. Just like in substitution, the idea is to eliminate one variable so that we can solve for the other. This is done by adding or subtracting equations in the system. Let's look at an example.

$$\begin{cases} 5x + 7y = 64 \\ 0.5x - 7y = \text{-}9 \end{cases}$$

Notice that one equation has $7y$ and the other has $\text{-}7y$.

If we add the second equation to the first, the $7y$ and $\text{-}7y$ add up to 0, which eliminates the y-variable, allowing us to solve for x.

$$\begin{aligned} 5x + 7y &= 64 \\ 0.5x - 7y &= \text{-}9 \quad + \\ \hline 5.5x + 0 &= 55 \\ 5.5x &= 55 \\ x &= 10 \end{aligned}$$

Now that we know $x = 10$, we can substitute 10 for x in either of the equations and find y:

$$\begin{aligned} 5x + 7y &= 64 \\ 5(10) + 7y &= 64 \\ 50 + 7y &= 64 \\ 7y &= 14 \\ y &= 2 \end{aligned}$$

$$\begin{aligned} 0.5x - 7y &= \text{-}9 \\ 0.5(10) - 7y &= \text{-}9 \\ 5 - 7y &= \text{-}9 \\ \text{-}7y &= \text{-}14 \\ y &= 2 \end{aligned}$$

In this system, the coefficient of y in the first equation happens to be the opposite of the coefficient of y in the second equation. The sum of the terms with y-variables is 0.

What if the equations don't have opposite coefficients for the same variable, like in the following system?

$$\begin{cases} 8r + 4s = 12 \\ 8r + s = \text{-}3 \end{cases}$$

Notice that both equations have $8r$, and if we subtract the second equation from the first, the variable r will be eliminated because $8r - 8r$ is 0.

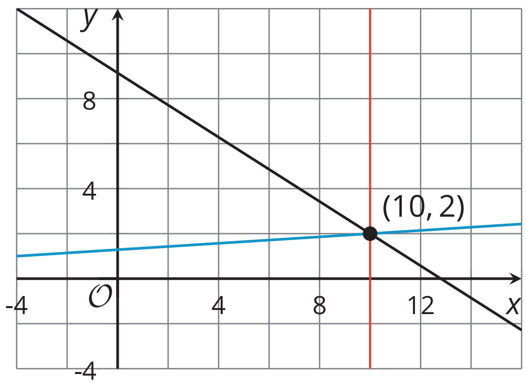
$$\begin{aligned} 8r + 4s &= 12 \\ 8r + s &= \text{-}3 \quad - \\ \hline 0 + 3s &= 15 \\ 3s &= 15 \\ s &= 5 \end{aligned}$$

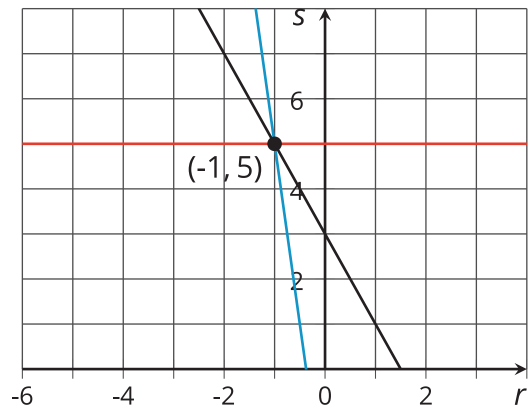
Substituting 5 for s in one of the equations gives us r:

$$\begin{aligned} 8r + 4s &= 12 \\ 8r + 4(5) &= 12 \\ 8r + 20 &= 12 \\ 8r &= \text{-}8 \\ r &= \text{-}1 \end{aligned}$$

Adding or subtracting the equations in a system creates a new equation. How do we know the new equation shares a solution with the original system?

If we graph the original equations in the system and the new equation, we can see that all three lines intersect at the same point, but why do they?





In future lessons, we will investigate why this strategy works.

Glossary

- elimination

Lesson 14 Practice Problems

1 Student Task Statement

Which equation is the result of adding these two equations?

$$\begin{cases} -2x + 4y = 17 \\ 3x - 10y = -3 \end{cases}$$

A. $-5x - 6y = 14$

B. $-x - 6y = 14$

C. $x - 6y = 14$

D. $5x + 14y = 20$

Solution

C

2 Student Task Statement

Which equation is the result of subtracting the second equation from the first?

$$\begin{cases} 4x - 6y = 13 \\ -5x + 2y = 5 \end{cases}$$

A. $-9x - 4y = 8$

B. $-x + 4y = 8$

C. $x - 4y = 8$

D. $9x - 8y = 8$

Solution

D

3 Student Task Statement

Solve this system of equations without graphing: $\begin{cases} 5x + 2y = 29 \\ 5x - 2y = 41 \end{cases}$

Solution

$(7, -3)$

4

Student Task Statement

Here is a system of linear equations: $\begin{cases} 6x + 21y = 103 \\ \text{-}6x + 23y = 51 \end{cases}$

Would you rather use subtraction or addition to solve the system? Explain your reasoning.

Solution

Sample response: I would rather use addition. Adding the two equations immediately eliminates x and allows us to solve for y.

5 from Unit 4, Lesson 8

Student Task Statement

Kiran sells f full boxes and h half-boxes of fruit to raise money for a band trip. He earns \$5 for each full box and \$2 for each half-box of fruit he sells and earns a total of \$100 toward the cost of his band trip. The equation $5f + 2h = 100$ describes this relationship.

Solve the equation for f.

Solution

Sample response: $f = \frac{(100 - 2h)}{5}$

6 from Unit 4, Lesson 8

Student Task Statement

Match each equation with the corresponding equation solved for a.

A. $a + 2b = 5$

B. $5a = 2b$

C. $a + 5 = 2b$

D. $5(a + 2b) = 0$

E. $5a + 2b = 0$

1. $a = \frac{2b}{5}$
2. $a = \frac{\text{-}2b}{5}$
3. $a = \text{-}2b$
4. $a = 2b - 5$
5. $a = 5 - 2b$

Solution

- A matches 5
- B matches 1

- C matches 4
- D matches 3
- E matches 2

7 from Unit 4, Lesson 9

Student Task Statement

The volume of a cylinder is represented by the formula $V = \pi r^2 h$.

Find each missing height and show your reasoning.

volume (cubic inches)	radius (inches)	height (inches)
96π	4	
31.25π	2.5	
V	r	

Solution

6, 5, $\frac{V}{\pi r^2}$ (heights in inches)

8 from Unit 4, Lesson 11

Student Task Statement

Match each equation with the slope m and y-intercept of its graph.

A. $m = \text{-}6$, $y\text{-int} = (0, 12)$

B. $m = \text{-}6$, $y\text{-int} = (0, 5)$

C. $m = \text{-}\frac{5}{6}$, $y\text{-int} = (0, 1)$

D. $m = \frac{5}{6}$, $y\text{-int} = (0, 1)$

E. $m = \frac{5}{6}$, $y\text{-int} = (0, -1)$

F. $m = \frac{5}{6}$, $y\text{-int} = (0, \text{-}5)$

1. $5x - 6y = 30$
2. $y = 5 - 6x$
3. $y = \frac{5}{6}x + 1$
4. $5x - 6y = 6$
5. $5x + 6y = 6$
6. $6x + y = 12$

Solution

- A matches 6

- B matches 2
- C matches 5
- D matches 3
- E matches 4
- F matches 1

9 from Unit 4, Lesson 13

Student Task Statement

Solve each system of equations.

a. $\begin{cases} 2x + 3y = 4 \\ 2x = 7y + 24 \end{cases}$

b. $\begin{cases} 5x + 3y = 23 \\ 3y = 15x - 21 \end{cases}$

Solution

a. $(5, \text{-}2)$
b. $(2.2, 4)$

10 from Unit 4, Lesson 13

Student Task Statement

Elena and Kiran are playing a board game. After one round, Elena says, "You earned so many more points than I did. If you earned 5 more points, your score would be twice mine!"

Kiran says, "Oh, I don't think I did that much better. I only scored 9 points higher than you did."

a. Write a system of equations to represent each student's comment. Be sure to specify what your variables represent.
b. If both students were correct, how many points did each student score? Show your reasoning.

Solution

a. Let k represent Kiran's score and e represent Elena’s score: $\begin{cases} k = e + 9 \\ k + 5 = 2e \end{cases}$

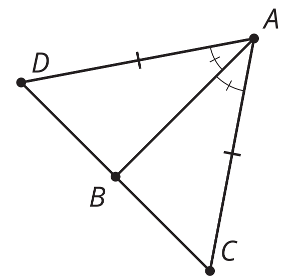
b. Kiran’s score is 23 and Elena’s score is 14. Sample reasoning: Substituting $e + 9$ for k in the second equation gives $e + 9 + 5 = 2e$ or $e + 14 = 2e$, so $e = 14$. Kiran's score is 9 points higher, so it's 23.

11 from Unit 2, Lesson 8

Student Task Statement

In isosceles triangle DAC, AD is congruent to AC, and AB is an angle bisector of angle DAC. How does Kiran know that AB is a perpendicular bisector of segment CD?

$\angle DAB \cong \angle CAB, \overline{AD} \cong \overline{AC}$



Solution

Sample response: Kiran has enough information to show that triangle ABD and triangle ABC are congruent by the Side-Angle-Side Triangle Congruence Theorem, so he knows that B is the midpoint of DC. Point A is the same distance from C as it is from D, so it must be on the perpendicular bisector of DC. Therefore, AB is a perpendicular bisector of segment CD.