Looking at Rates of Change

Let's calculate average rates of change for exponential functions.

13.1 Falling Prices

Let p be the function that gives the cost $p(t)$, in dollars, of producing 1 watt of solar power t years after 1977. Here is a table showing the values of p from 1977 to 1987.

t	$p(t)$
0	80
1	60
2	45
3	33.75
4	25.31
5	18.98
6	14.24
7	10.68
8	8.01
9	6.01
10	4.51

Which expression can be used to calculate the average rate of change in solar cost between 1977 and 1987?

A. $p(10) - p(0)$

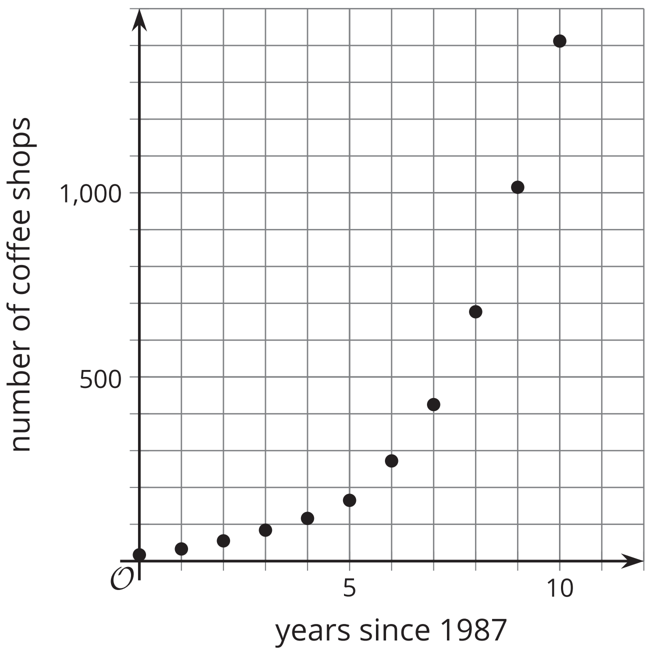
B. $p(10)$

C. $\frac{p(10) - p(0)}{10 - 0}$

D. $\frac{p(10)}{p(0)}$

13.2 Coffee Shops

Here are a table and a graph that show the number of coffee shops worldwide that a company had in its first 10 years, between 1987 and 1997. The growth in the number of stores was roughly exponential.



year	number of stores
1987	17
1988	33
1989	55
1990	84
1991	116
1992	165
1993	272
1994	425
1995	677
1996	1,015
1997	1,412

1. Find the average rate of change for each period of time. Show your reasoning.

   a. 1987 and 1990

   b. 1987 and 1993

   c. 1987 and 1997

2. Make some observations about the rates of change that you calculated. What do these average rates tell us about how the company was growing during this time period?

3. Use the graph to support your answers to these questions. How well do the average rates of change describe the growth of the company in:

   a. the first 3 years?

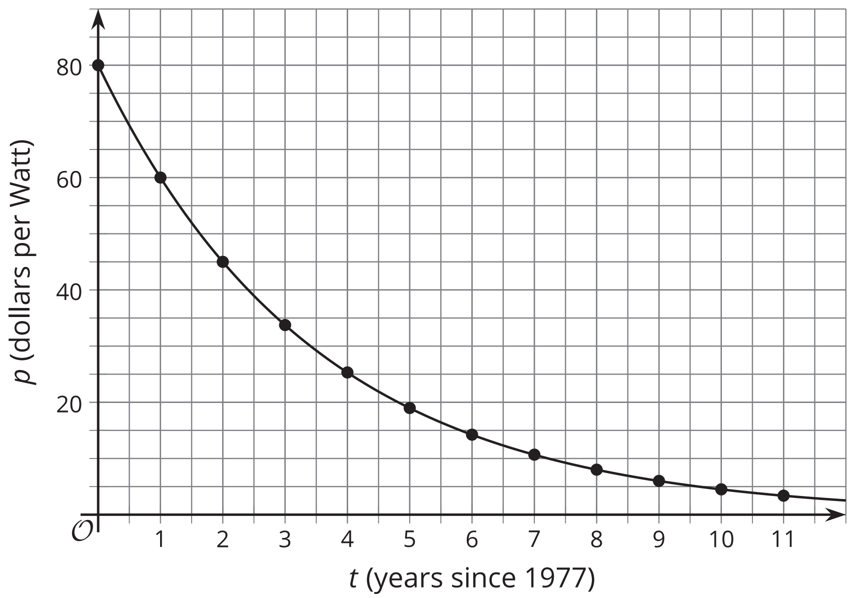
   b. the first 6 years?

   c. the entire 10 years?

4. Let f be the function so that $f(t)$ represents the number of stores t years since 1987. The value of $f(20)$ is 15,011. Find $\frac{f(20) - f(10)}{20 - 10}$, and say what it tells us about the change in the number of stores.

13.3 Revisiting Cost of Solar Cells

This graph represents the exponential function, p, which models the cost $p(t)$, in dollars, of producing 1 watt of solar energy, from 1977 to 1988, where t is years since 1977.



1. Clare said, "In the first five years, between 1977 and 1982, the cost fell by about $12 per year. But in the second five years, between 1983 and 1988, the cost fell only by about $2 a year." Show that Clare is correct.

2. If the trend continues, will the average decrease in price be more or less than $2 per year between 1987 and 1992? Explain your reasoning.

Are you ready for more?

Suppose the cost of producing 1 watt of solar energy had instead decreased by $12.20 each year between 1977 and 1982. Compute what the costs would be each year and plot them on the same graph shown in the activity. How do these alternate costs compare to the actual costs shown?

Lesson 13 Summary

When we calculate the average rate of change for a linear function, no matter what interval we pick, the value of the rate of change is the same. A constant rate of change is an important feature of linear functions! When a linear function is represented by a graph, the slope of the line is the rate of change of the function.

Exponential functions also have important features. We've learned about exponential growth and exponential decay, both of which are characterized by a constant quotient over equal intervals. But what does this mean for the value of the average rate of change for an exponential function over a specific interval?

Let's look at an exponential function that we studied earlier. Let A be the function that models the area, $A(t)$, in square yards, of algae covering a pond t weeks after beginning treatment to control the algae bloom. Here is a table showing about how many square yards of algae remain during the first 5 weeks of treatment.

t	$A(t)$
0	240
1	80
2	27
3	9
4	3

The average rate of change of A from the start of treatment to Week 2 is about -107 square yards per week because $\frac{A(2) - A(0)}{2 - 0} \approx -107$. The average rate of change of A from Week 2 to Week 4, however, is only about -12 square yards per week because $\frac{A(4) - A(2)}{4 - 2} \approx -12$.

The negative average rates of change show that A is decreasing over both intervals, but the average rate of change for the time during Weeks 0 to 2 indicates that the values are decreasing more rapidly than during Weeks 2 to 4 due to the effect of the decay factor. For an exponential function with a growth factor greater than 1, the values for the average rate of change of each interval are positive, with the second interval increasing more quickly due to the effect of the growth factor.