Unit 4, Lesson 14

More Arithmetic with Complex Numbers

Let’s practice adding, subtracting, and multiplying complex numbers.

14.1 Which Three Go Together: Complex Expressions

Which three go together? Why do they go together?

A

$$2i \cdot i$$

B

$$(1 + i) + (1 - i)$$

C

$$(1 + i)^2$$

D

$$(1 + 2i)(1 - 2i)$$

14.2 Powers of i

1. Write each power of i in the form $a + bi$, where a and b are real numbers. If a or b is zero, you do not need to write that part of the number. For example, $0 + 3i$ can be expressed as $3i$.

   i^0 i^1 i^2

   i^3 i^4 i^5

   i^6 i^7 i^8

2. Use any patterns you noticed to rewrite i^{100} in a similar way. Explain your reasoning.

3. Use any patterns you noticed to rewrite i^{38} in a similar way. Explain your reasoning.

Are you ready for more?

1. Write each power of $1 + i$ in the form $a + bi$, where a and b are real numbers. If a or b is zero, you do not need to write that part of the number. For example, $0 + 3i$ can be expressed as $3i$.

   $(1 + i)^0$ $(1 + i)^1$ $(1 + i)^2$

   $(1 + i)^3$ $(1 + i)^4$ $(1 + i)^5$

   $(1 + i)^6$ $(1 + i)^7$ $(1 + i)^8$

2. Compare and contrast the powers of $1 + i$ with the powers of i when plotted on the complex plane. What is the same? What is different?

14.3 Add 'Em Up (or Subtract or Multiply)

For each row, your partner and you will each rewrite an expression so it has the form $a + bi$, where a and b are real numbers. You and your partner should get the same answer. If you disagree, work to reach an agreement.

partner A	partner B
$(7 + 9i) + (3 - 4i)$	$5i(1 - 2i)$
$2i(3 + 4i)$	$(1 + 2i) - (9 - 4i)$
$(4 - 3i)(4 + 3i)$	$(5 + i) + (20 - i)$
$(2i)^4$	$(3 + i\sqrt{7})(3 - i\sqrt{7})$
$(1 + i\sqrt{5}) - (\text{-}7 - i\sqrt{5})$	$(\text{-}2i)(\text{-}\sqrt{5} + 4i)$
$\left(\frac{1}{2}i\right)\left(\frac{1}{3}i\right)\left(\frac{3}{4}i\right)$	$\left(\frac{1}{2}i\right)^3$

Lesson 14 Summary

Suppose we want to write the product $(3+5i)(7-2i)$ in the form $a+bi$, where a and b are real numbers. For example, we might want to compare our solution with a partner's, and having answers in the same form makes that easier. Using the distributive property,

$$\begin{aligned}(3+5i)(7-2i) &= 21-6i+35i-10i^2\\ &= 21+29i-10(\text{-}1)\\ &= 21+29i+10\\ &= 31+29i\end{aligned}$$

Keeping track of the negative signs is especially important since it is easy to mix up the fact that $i^2 = \text{-}1$ with the fact that $\text{-}i^2 = \text{-}(\text{-}1) = 1$.

Next, suppose we want to write the difference $(\text{-}6+3i)-(2-4i)$ as a single complex number in the form $a+bi$. Distributing the negative and combining like terms, we get:

$$\begin{aligned}(\text{-}6+3i)-(2-4i) &= \text{-}6+3i-2-(\text{-}4i)\\ &= \text{-}8+3i+4i\\ &= \text{-}8+7i\end{aligned}$$

Again, it is important to be precise with negative signs. It is a common mistake to subtract $4i$ rather than subtracting $\text{-}4i$.