

Unit 2 Lesson 19: End Behavior of Rational Functions

1 Different Divisions, Revisited (Warm up)

Student Task Statement

Complete all three representations of the polynomial division following the forms of the integer division.

$$\begin{array}{r}
 252 \\
 11 \overline{) 2775} \\
 \underline{2200} \\
 575 \\
 \underline{550} \\
 25 \\
 \underline{22} \\
 3
 \end{array}$$

$$2775 = 11(252) + 3$$

$$\frac{2775}{11} = 252 + \frac{3}{11}$$

$$\begin{array}{r}
 2x^2 \\
 x+1 \overline{) 2x^3 + 7x^2 + 7x + 5}
 \end{array}$$

$$2x^3 + 7x^2 + 7x + 5 =$$

$$\frac{2x^3 + 7x^2 + 7x + 5}{x+1} =$$

2 Combined Fuel Economy

Student Task Statement

In 2000, the Environmental Protection Agency (EPA) reported a combined fuel efficiency for cars that assumes 55% city driving and 45% highway driving. The expression for the combined fuel efficiency of a car that gets x mpg in the city and h mpg on the highway can be written as $\frac{100xh}{55x+45h}$.

1. Several conventional cars have a fuel economy for highway driving that is about 10 mpg higher than for city driving. That is, $h = x + 10$. Write a function f that represents the combined fuel efficiency for cars like these in terms of x .
2. Rewrite f in the form $q(x) + \frac{r(x)}{b(x)}$ where $q(x)$, $r(x)$, and $b(x)$ are polynomials.

3 Exploring End Behavior

Student Task Statement

function	degree of num.	degree of den.	rewritten in the form of $q(x) + \frac{r(x)}{b(x)}$	end behavior
$g(x) = -\frac{5}{x+2}$				
$h(x) = \frac{7x-5}{x+2}$				
$j(x) = \frac{3x^2+7x-5}{x+2}$				
$k(x) = \frac{2x^3+3x^2+7x-5}{x+2}$				
$m(x) = \frac{x+2}{2x^3+3x^2+7x-5}$				

1. Complete the table to explore the end behavior for rational functions.
2. What do you notice about the end behavior of different types of rational functions?