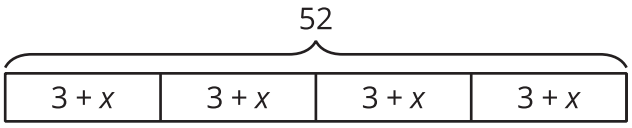
Unit 6 Family Support Materials

Expressions, Equations, and Inequalities

Section A: Representing Situations of the Form $px + q = r$ and $p(x + q) = r$

This week your student will be representing situations with diagrams and equations. We will look at two main types of situations.

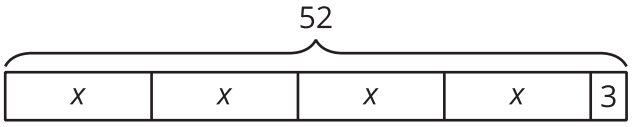
Here is an example of the first type of situation: A standard deck of playing cards has four suits. In each suit, there are 3 face cards and x other cards. There are 52 total cards in the deck.



$$52 = 4(3 + x)$$

The diagram and the equation both show that there are 4 groups of cards, each group contains $x + 3$ cards, and there are 52 cards in all.

Here is an example of the second type of situation: A chef makes 52 pints of spaghetti sauce. She sets aside 3 pints to take home to her family. She divides the rest of the sauce equally into 4 containers.



$$52 = 4x + 3$$

The diagram and the equation both show that from the 52 total pints, 3 were set aside, and each of 4 containers holds x pints of sauce.

Here is a task to try with your student:

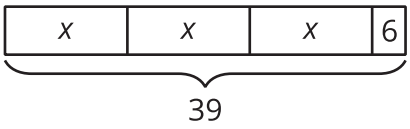
1. Draw a diagram to represent the equation $3x + 6 = 39$.
2. Draw a diagram to represent the equation $39 = 3(y + 6)$.
3. Decide which story goes with which equation and diagram:
   - Three friends went cherry picking, and each picked the same amount of cherries, in pounds. Before they left the cherry farm, someone gave them an additional 6 pounds of cherries. Altogether, they had 39 pounds of cherries.

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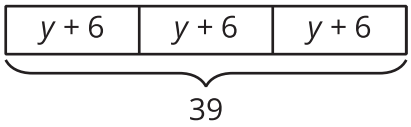
- One of the friends made three cherry tarts. She put the same number of cherries in each tart, and then added 6 more cherries to each tart. Altogether, the three tarts contained 39 cherries.

Solution:

1. **A**



2. **B**



3. Diagram A and $3x + 6 = 39$ represent the story about cherry picking. Diagram B and $3(y + 6) = 39$ represent the story about making cherry tarts.

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Section B: Solving Equations of the Form $px + q = r$ and $p(x + q) = r$ and Problems That Lead to Those Equations

This week your student will be solving equations. Sometimes to solve an equation, we can just think of a number that would make the equation true. For example, the solution to $12 - c = 10$ is 2, because we know that $12 - 2 = 10$. For more complicated equations that may include decimals, fractions, and negative numbers, the solution may not be so obvious.

An important method for solving equations is *doing the same thing to each side*. For example, let's see how we might solve $-4(x - 1) = 24$ by doing the same thing to each side.

$$\begin{aligned} -4(x-1) &= 24 \\ -\tfrac{1}{4} \cdot -4(x-1) &= -\tfrac{1}{4} \cdot 24 && \text{Multiply each side by } -\tfrac{1}{4}. \\ x - 1 &= -6 \\ x - 1 + 1 &= -6 + 1 && \text{Add 1 to each side.} \\ x &= -5 \end{aligned}$$

Another helpful tool for solving equations is to apply the distributive property. For the above example, if we start by using the distributive property, then the solution would look like this:

$$\begin{aligned} -4(x-1) &= 24 \\ -4x + 4 &= 24 && \text{Apply the distributive property.} \\ -4x + 4 - 4 &= 24 - 4 && \text{Subtract 4 from each side.} \\ -4x &= 20 \\ -4x \div -4 &= 20 \div -4 && \text{Divide each side by } -4. \\ x &= -5 \end{aligned}$$

Either method shows us that the equation $-4(x - 1) = 24$ is true when $x = -5$.

Here is a task to try with your student:

Elena picks a number, adds 45 to it, and then multiplies by $\frac{1}{2}$. The result is 29. Elena says that you can find her number by solving the equation $29 = \frac{1}{2}(x + 45)$.

Find Elena's number. Describe the steps you used.

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Solution:

Elena's number is 13. There are many different ways to solve her equation. Here is one example:

$$
\begin{aligned}
29 &= \frac{1}{2}(x + 45) \\
2 \cdot 29 &= 2 \cdot \frac{1}{2}(x + 45) && \text{Multiply each side by 2.} \\
58 &= x + 45 \\
58 - 45 &= x + 45 - 45 && \text{Subtract 45 from each side.} \\
13 &= x
\end{aligned}
$$

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Section C: Inequalities

This week your student will be working with inequalities (expressions with the symbol $>$ or $<$ instead of $=$). We use inequalities to describe a range of numbers. For example, in many places you need to be at least 16 years old to be allowed to drive. We can represent this situation with the inequality $a \geq 16$. We can show all the solutions to this inequality on the number line.



Here is a task to try with your student:

Noah already has \$10.50, and he earns \$3 each time he runs an errand for his neighbor. Noah wants to know how many errands he needs to run to have at least \$30, so he writes this inequality:

$$3e + 10.50 \geq 30$$

We can test this inequality for different values of e. For example, 4 errands is not enough for Noah to reach his goal, because $3 \cdot 4 + 10.50 = 22.5$, and \$22.50 is less than \$30.

1. Will Noah reach his goal if he runs:
   a. 8 errands?
   b. 9 errands?
2. What value of e makes the equation $3e + 10.50 = 30$ true?
3. What does this tell you about all the solutions to the inequality $3e + 10.50 \geq 30$?
4. What does this mean for Noah's situation?

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Solution:

1.

   a. Yes, if Noah runs 8 errands, he will have $3 \cdot 8 + 10.50$, or $34.50.

   b. Yes, since 9 is more than 8, and 8 errands was enough, 9 will also be enough.

2. The equation is true when $e = 6.5$. We can rewrite the equation as $3e = 30 - 10.50$, or $3e = 19.50$. Then we can rewrite this as $e = 19.50 \div 3$, or $e = 6.5$.

3. This means that when $e \geq 6.5$, Noah's inequality is true.

4. Noah can't really run 6.5 errands, but he could run 7 or more errands, and then he would have more than $30.

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Section D: Writing Equivalent Expressions

This week your student will be working with equivalent expressions (expressions that are always equal, for any value of the variable). For example, $2x + 7 + 4x$ and $6x + 10 - 3$ are equivalent expressions. We can see that these expressions are equal when we try different values for x.

	$2x + 7 + 4x$	$6x + 10 - 3$
when x is 5	$2 \cdot 5 + 7 + 4 \cdot 5$ $10 + 7 + 20$ 37	$6 \cdot 5 + 10 - 3$ $30 + 10 - 3$ 37
when x is -1	$2 \cdot -1 + 7 + 4 \cdot -1$ $-2 + 7 + -4$ 1	$6 \cdot -1 + 10 - 3$ $-6 + 10 - 3$ 1

We can also use properties of operations to see why these expressions have to be equivalent—they are each equivalent to the expression $6x + 7$.

Here is a task to try with your student:

Match each expression with an equivalent expression from the list below. One expression in the list will be left over.

1. $5x + 8 - 2x + 1$
2. $6(4x - 3)$
3. $(5x + 8) - (2x + 1)$
4. $-12x + 9$

List:

- $3x + 7$
- $3x + 9$
- $-3(4x - 3)$
- $24x + 3$
- $24x - 18$

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Solution:

1. $3x + 9$ is equivalent to $5x + 8 - 2x + 1$, because $5x + -2x = 3x$ and $8 + 1 = 9$.
2. $24x - 18$ is equivalent to $6(4x - 3)$, because $6 \cdot 4x = 24x$ and $6 \cdot -3 = -18$.
3. $3x + 7$ is equivalent to $(5x + 8) - (2x + 1)$, because $5x - 2x = 3x$ and $8 - 1 = 7$.
4. $-3(4x - 3)$ is equivalent to $-12x + 9$, because $-3 \cdot 4x = -12x$ and $-3 \cdot -3 = 9$.

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