Unit 3 Family Support Materials

Fractions and Decimals

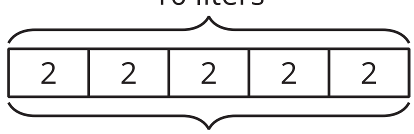
Section A: Making Sense of Division

This week, your student will be thinking about the meaning of division to prepare to learn about division of fractions. Suppose we have 10 liters of water to divide into equal-size groups. We can think of the division $10 \div 2$ in two ways, or as the answer to two questions:

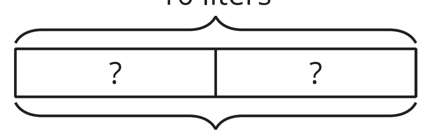
- "How many bottles can we fill with 10 liters if each bottle has 2 liters?"
- "How many liters are in each bottle if we divide 10 liters into 2 bottles?"

Here are two diagrams to show the two interpretations of $10 \div 2$:

A



B



In both cases, the answer to the question is 5, but it could mean either "there are *5 bottles* with 2 liters in each" or "there are *5 liters* in each of the 2 bottles."

Here is a task to try with your student:

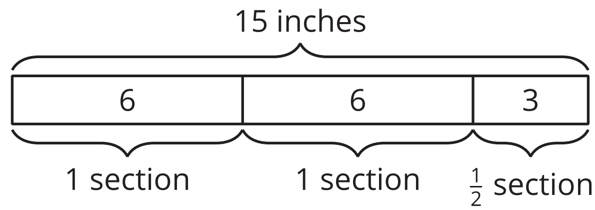
1. Write two different questions we can ask about $15 \div 6$.
2. Estimate the answer: Is it less than 1, equal to 1, or greater than 1? Explain your estimate.
3. Find the answer to one of the questions you wrote. It might help to draw a picture.

Solution:

1. Sample questions:
    - A ribbon that is 15 inches long is divided into 6 equal sections. How long (in inches) is each section?
    - A ribbon that is 15 inches is divided into 6-inch sections. How many sections are there?
2. Greater than 1. Sample reasoning:
    - $12 \div 6$ is 2, so $15 \div 6$ must be greater than 2.
    - If we divide 15 into 15 groups ($15 \div 15$), we get 1. So if we divide 15 into 6, which is a

smaller number of groups, the amount in each group must be greater than 1.

3. $2\frac{1}{2}$. Sample diagram:



Section B: Dividing Fractions

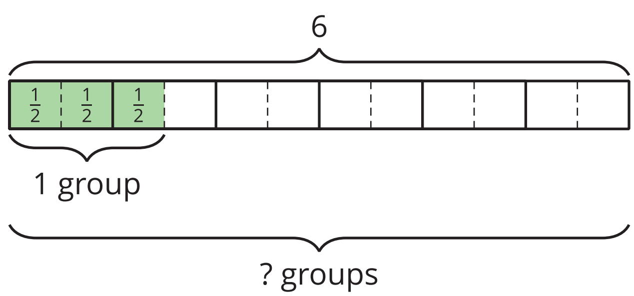
Earlier, students learned that a division such as $10 \div 2 = ?$ can be interpreted as "*How many groups* of 2 are in 10?" or "*How much is in each group* if there are 10 in 2 equal-size groups?" They also saw that the relationship between 10, 2, and the unknown number ("?") can also be expressed with multiplication:

$$2 \cdot ? = 10$$

$$? \cdot 2 = 10$$

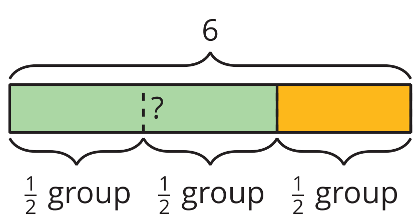
This week, they use these ideas to divide fractions. For example, $6 \div 1\frac{1}{2} = ?$ can be thought of as "How many groups of $1\frac{1}{2}$ are in 6?" Expressing the question as a multiplication and drawing a diagram can help us find the answer.

$$? \cdot 1\frac{1}{2} = 6$$



From the diagram we can count that there are 4 groups of $1\frac{1}{2}$ in 6.

We can also think of $6 \div 1\frac{1}{2} = ?$ as "How much is in each group if there are $1\frac{1}{2}$ equal groups in 6?" A diagram can also be useful here.



From the diagram we can see that there are three $\frac{1}{2}$ groups in 6. This means there is 2 in each $\frac{1}{2}$ group, or 4 in 1 group.

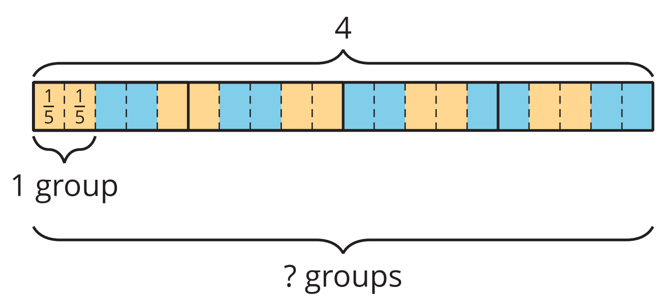
In both cases, $6 \div 1\frac{1}{2} = 4$, but the 4 can mean different things depending on how the division is interpreted.

Here is a task to try with your student:

1. How many groups of $\frac{2}{3}$ are in 5?
    a. Write a division equation to represent the question. Use a "?" to represent the unknown amount.
    b. Find the answer. Explain or show your reasoning.
2. A sack of flour weighs 4 pounds. A grocer is distributing the flour into equal-size bags.
    a. Write a question such that $4 \div \frac{2}{5} = ?$ could represent the situation.
    b. Find the answer. Explain or show your reasoning.

Solution:

1. a. $5 \div \frac{2}{3} = ?$
    b. $7\frac{1}{2}$. Sample reasoning: There are 3 thirds in 1, so there are 15 thirds in 5. That means there are half as many two-thirds, or $\frac{15}{2}$ two-thirds, in 5.
2. a. 4 pounds of flour are divided equally into bags of $\frac{2}{5}$ pound each. How many bags will there be?
    b. 10 bags. Sample reasoning: Break every 1 pound into fifths, and then count how many groups of $\frac{2}{5}$ there are.

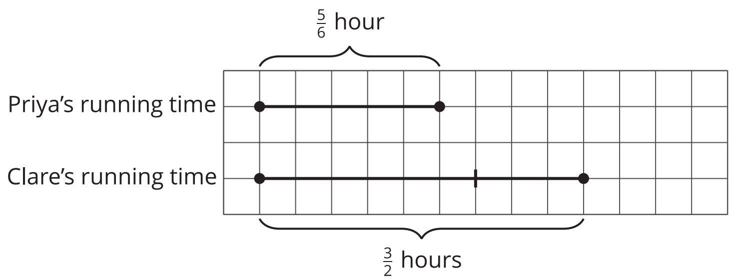


Section C: Fractions in Lengths, Areas, and Volumes

Over the next few days, your student will be solving problems that require multiplying and dividing fractions. Some of these problems will be about comparison. For example:

- If Priya ran for $\frac{5}{6}$ hour and Clare ran for $\frac{3}{2}$ hours, what fraction of Clare's running time was Priya's running time?

  We can draw a diagram and write a multiplication equation to make sense of the situation.



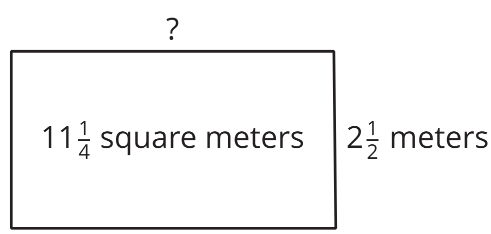
$$\text{(fraction)} \cdot \text{(Clare's time)} = \text{(Priya's time)}$$

$$? \cdot \frac{3}{2} = \frac{5}{6}$$

  We can find the unknown by dividing. $\frac{5}{6} \div \frac{3}{2} = \frac{5}{6} \cdot \frac{2}{3}$, which equals $\frac{10}{18}$. So Priya's running time was $\frac{10}{18}$, or $\frac{5}{9}$, of Clare's.

Other problems your students will solve are related to geometry—lengths, areas, and volumes. Here are some examples:

- What is the length of a rectangular room if its width is $2\frac{1}{2}$ meters and its area is $11\frac{1}{4}$ square meters?



  We know that the area of a rectangle can be found by multiplying its length and width ($? \cdot 2\frac{1}{2} = 11\frac{1}{4}$), so dividing $11\frac{1}{4} \div 2\frac{1}{2}$ (or $\frac{45}{4} \div \frac{5}{2}$) will give us the length of the room. $\frac{45}{4} \div \frac{5}{2} = \frac{45}{4} \cdot \frac{2}{5} = \frac{9}{2}$. The room is $4\frac{1}{2}$ meters long.

- What is the volume of a box (a rectangular prism) that is $3\frac{1}{2}$ feet by 10 feet by $\frac{1}{4}$ foot?

  We can find the volume by multiplying the edge lengths. $3\frac{1}{2} \cdot 10 \cdot \frac{1}{4} = \frac{7}{2} \cdot 10 \cdot \frac{1}{4}$, which equals $\frac{70}{8}$. So the volume is $\frac{70}{8}$, or $8\frac{6}{8}$, cubic feet.

Here is a task to try with your student:

1. In the first example about Priya's and Clare's running times, how many times as long as

Priya’s running time was Clare’s running time? Show your reasoning.

2. The area of a rectangle is $\frac{20}{3}$ square feet. What is its width if its length is $\frac{4}{3}$ feet? Show your reasoning.

Solution:

1. $\frac{9}{5}$. Sample reasoning: We can write $? \cdot \frac{5}{6} = \frac{3}{2}$ to represent the question “How many times of Priya’s running time was Clare’s running time?” and then solve by dividing: $\frac{3}{2} \div \frac{5}{6} = \frac{3}{2} \cdot \frac{6}{5} = \frac{18}{10}$. Clare’s running time was $\frac{18}{10}$, or $\frac{9}{5}$, as long as Priya’s.
2. 5 feet. Sample reasoning: $\frac{20}{3} \div \frac{4}{3} = \frac{20}{3} \cdot \frac{3}{4} = \frac{20}{4} = 5$

Section D: Adding, Subtracting, and Multiplying Decimals

This week, your student will add, subtract, and multiply numbers using what they know about the meaning of the digits.

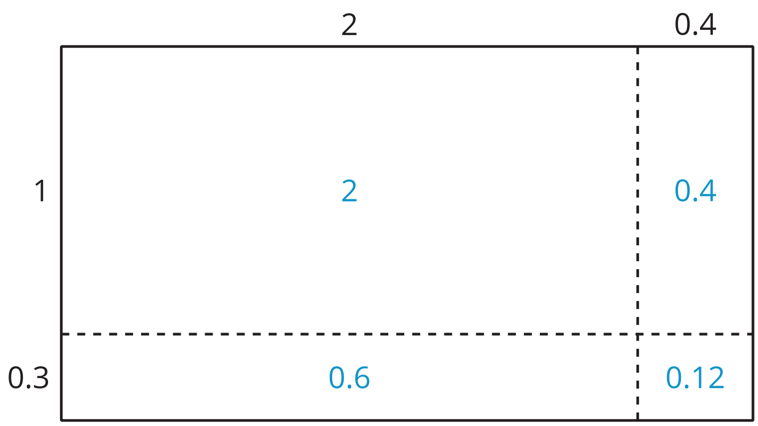
To add whole numbers and decimal numbers, we can arrange $0.921 + 4.37$ vertically, aligning the decimal points, and find the sum. This is a convenient way to be sure we are adding digits that correspond to the same units. This also makes it easy to keep track when we compose 10 units into the next higher unit. (Some people call this "carrying.")

$$\begin{array}{r@{}l} \overset{1}{0}&.921 \\ +\ 4&.37 \\ \hline 5&.291 \end{array}$$

There are a few ways we can multiply two decimals such as $(2.4) \cdot (1.3)$. One way is to represent the product as the area of a rectangle. If 2.4 and 1.3 are the side lengths of a rectangle, the product of $(2.4) \cdot (1.3)$ is its area.

To find the area, it helps to decompose the rectangle into smaller rectangles by breaking the side lengths apart by place value. In this case, 2.4 can be decomposed into 2 and 0.4, and 1.3 can be decomposed into 1 and 0.3.

Then we can find the area of each smaller rectangle. The sum of the areas of all of the smaller rectangles, 3.12, is the total area.



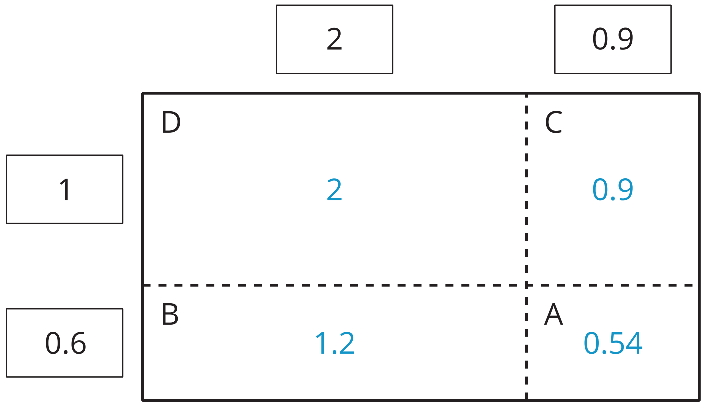
$$\begin{array}{r@{}l} 0&.12 \\ 0&.6 \\ 0&.4 \\ +\ 2& \\ \hline 3&.12 \end{array}$$

Here is a task to try with your student:

Find $(2.9) \cdot (1.6)$ using an area model and partial products.

Solution: 4.64. The area of the rectangle (or the sum of the partial products) is

$2 + 0.9 + 1.2 + 0.54 = 4.64$



Section E: Dividing Decimals

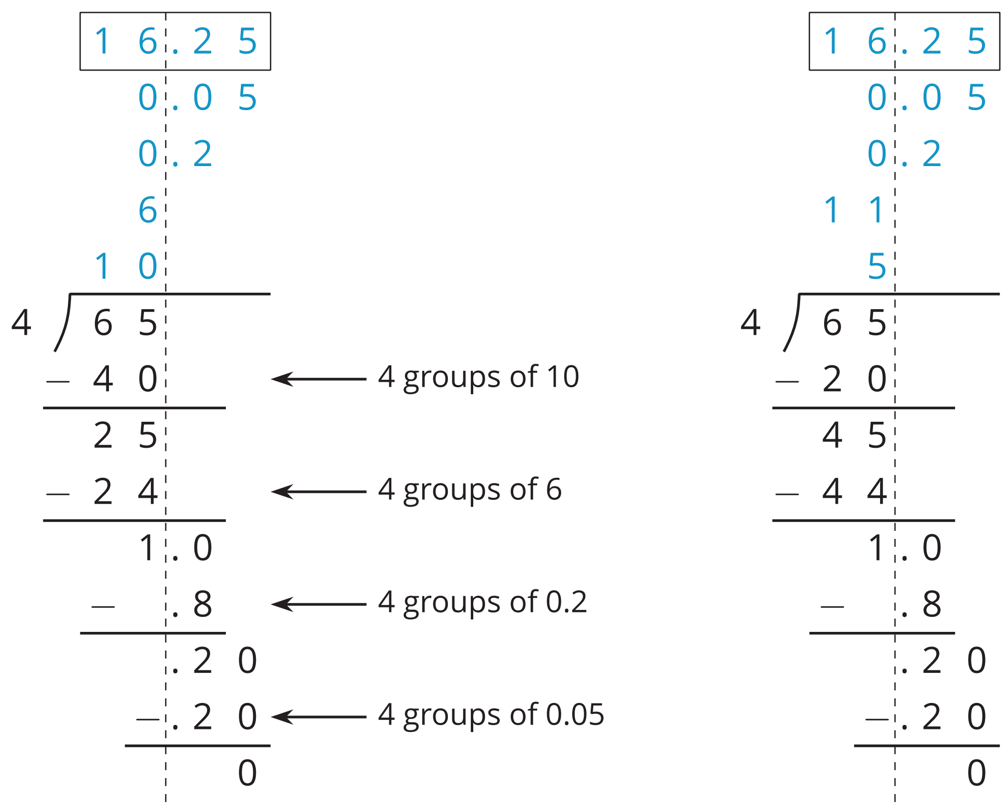
This week, your student will divide whole numbers and decimals. We can think about division as breaking apart a number into equal-size groups.

Let's take $65 \div 4$ for example. We can imagine that we are sharing 65 dollars equally among 4 people. Here is one way to think about this:

- First, give each person 10 dollars. This means 40 dollars are shared out, and 25 dollars are left over.
- Next, give each person 6 more dollars. This means 24 more dollars are shared out, and 1 dollar is left.
- Then, give each person another 0.2 of a dollar. This means 0.8 of a dollar is shared out and 0.2 of a dollar is left.
- Finally, give each person 0.05 of a dollar. There is no money left.

The 65 dollars are divided into 4 equal groups. Everyone gets $10 + 6 + 0.2 + 0.05$, or 16.25, dollars.

The calculation on the left shows one way to record these steps for dividing.



The calculation on the right shows different intermediate steps, but the quotient is the same. We say that this method of dividing uses *partial quotients*.

Here is a task to try with your student:

$$
\begin{array}{r}
\boxed{112} \\
2 \\
10 \\
100 \\
7 \enclose{longdiv}{784} \\
-\ 700 \\
\hline
84 \\
-\ 70 \\
\hline
14 \\
-\ 14 \\
\hline
0
\end{array}
$$

Here is how Jada found $784 \div 7$ using partial quotients.

1. In the calculation, what does the subtraction of 700 represent?
2. Above the dividend 784, we see the numbers 100, 10, and 2. What do they represent?
3. How can we check if 112 if the correct quotient for $784 \div 7$?

Solution

1. It represents the subtraction of 7 groups of 100 from 784.
2. 100, 10, and 2 are the amounts distributed into each group over 3 rounds of dividing. There are 7 groups of 100 in 700, 7 groups of 10 in 70, and 7 groups of 2 in 14.
3. We can multiply $7 \cdot 112$ and see if it produces 784.