Scope and Sequence for Integrated Math 3

Students begin the course with a study of solid geometry. They examine cross-sections of solids and make connections between cross-sections and dilations. Students also use square root and cube root graphs to illustrate the relationship between scale factors and scaled area and volume. This work is an opportunity to revisit functions and how they can be represented in a variety of ways.

This work leads students to analyze situations that are well modeled by polynomial functions before pivoting to study the structure of polynomial graphs and equations. Students do arithmetic on polynomial and rational functions and use different forms of the functions to identify asymptotes and end behavior. Students solve rational and radical equations and learn to recognize when the steps used to solve these types of equations result in solutions that are not solutions to the original equation. Students also study polynomial identities and use some key identities to establish the formula for the sum of the first n terms of a geometric sequence.

Students then return to their study of exponential functions and establish that the property of growth by equal factors over equal intervals holds even when the interval has non-integer length. Students use logarithms to solve for unknown exponents, and are introduced to the number e and its use in modeling continuous growth. Logarithm functions and some situations they model well are also briefly addressed.

Students learn to transform functions graphically and algebraically. In previous courses and units, students adjusted the parameters of particular types of models to fit data. In this part of the course, students consolidate and generalize this understanding. This work is useful in the study of periodic functions that comes next. Students work with the unit circle to make sense of trigonometric functions, and then students use trigonometric functions to model periodic relationships.

The last unit, on statistical inference, focuses on analyzing experimental data modeled by normal distributions. Students learn to use sampling and simulations to account for variability in data and estimate population mean, margin of error, and proportions. Students develop skepticism about news stories that summarize data inappropriately.

Modeling prompts are provided for use throughout the course. While students have opportunities to engage in aspects of mathematical modeling during class activities, modeling prompts allow students to engage in the full modeling cycle. Modeling prompts can be implemented in various ways. Please see the Mathematics Modeling Prompts section of this Course Guide for a more detailed explanation.

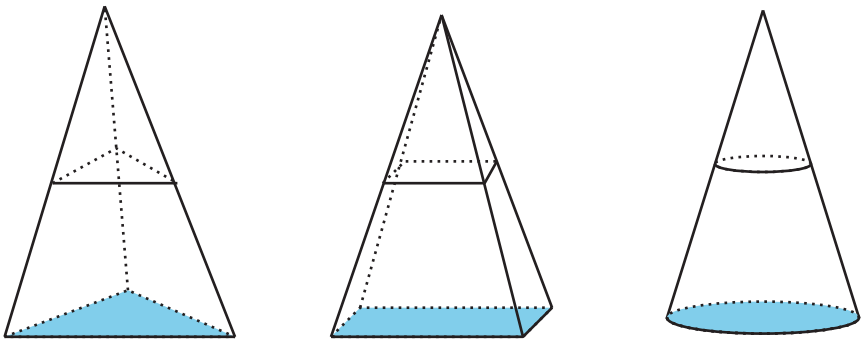
Unit 1: Solid Geometry

In this unit, students practice spatial visualization in three dimensions. They sketch cross-sections, study dilations, derive volume formulas, and apply their understandings to solve problems.

In previous grades, students studied multiple aspects of solids. In grade 6, they started calculating surface areas and volumes of right rectangular prisms. They extended this understanding to the volume and surface area of right prisms in grade 7, and to that of spheres, cones, and cylinders in grade 8. Students also described cross-sections of three-dimensional figures in grade 7.

In future courses, the visualization skills developed in this unit will be applicable when using calculus to compute volumes or when using linear algebra in three or more dimensions.

Students begin the unit by examining solids of rotation and cross-sections of a variety of solids. They make a connection between cross-sections and dilation to see that cross-sections of a pyramid may be viewed as dilations of the base for scale factors between 0 and 1. Later in the unit they learn Cavalieri's Principle: Suppose two solids have equal heights. If at all distances from the base, the cross-sections of the two solids have equal areas, then the solids have equal volumes. Combining the concepts of dilations, cross-sections, and Cavalieri's Principle with dissection allows students to derive the volume formulas for a pyramid or cone.



When students establish that dilating by a scale factor of k multiplies areas by k^2 and volumes by k^3, it provides an opportunity to use square root and cube root graphs to illustrate the relationship between scaled area or volume and scale factors. While this unit is the primary opportunity to study root functions, students will continue to graph functions and interpret the meaning of points, coefficients, and constants in future units.

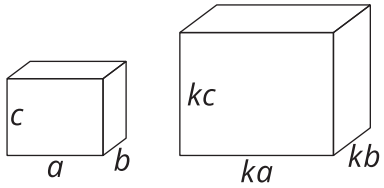
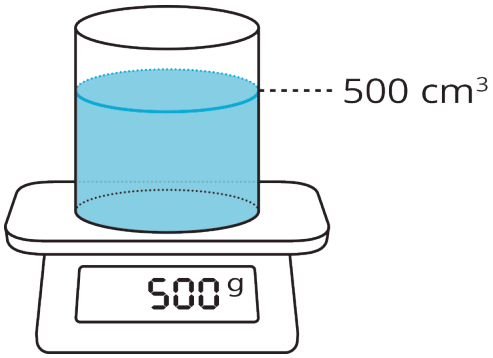
In this unit, students may assume that cylinders or prisms that appear to be oblique are indeed oblique, and those that appear to be right are right. For right cylinders and prisms, right angles will not be marked to indicate that the bases are at right angles to the lateral surfaces. Unless otherwise stated, all responses given in decimal form are rounded to the nearest tenth.

Geometry Reference Chart

In order to write convincing arguments, students need to support their statements with facts. The reference chart is a way to keep track of those facts for future reference when students are trying to prove new facts. At the beginning of the course, students are provided a chart with useful definitions, assertions, and theorems from previous courses in this sequence. Students continue adding entries and referring to them in the geometry sections of this course.

Print charts double sided to save paper. There should be a system for students to keep track of their charts (for example, hole punch and keep in a binder, or staple and tuck in the front of a notebook or the back of the workbook).

Each entry includes a statement, a diagram, a type and the date. A statement can be one of these three types: assertion, definition, or theorem. An assertion is an observation that seems to be true but is not proven. Sometimes assertions are not proven, because they are axioms or because the proof is beyond the scope of this course. The chart includes the most essential definitions. If there are additional definitions from this or previous courses that students would benefit from, feel free to add them. For example, it is assumed that students recall the definition of "isosceles." If this is not the case, that would be a useful definition to record. Here are some entries to show the chart's structure:

date, type	statement	diagram
[date] theorem	When any solid is dilated using a scale factor of k, all lengths are multiplied by k, all areas are multiplied by k^2, and all volumes are multiplied by k^3.	
[date] definition	The **density** of a substance is the mass of the substance per unit volume. That is, $\text{density} = \frac{\text{mass}}{\text{volume}}$.	**density: 1 gram per cm^3** 

Students are not expected to record all of their observations in the chart. Sometimes students' conjectures will be proven in a subsequent lesson and added later as theorems rather than assertions. Other times students prove something that they will not need to use again. Students are welcome to use any proven statement in a later proof, but the reference chart is designed to be as concise as possible so it is a more useful reference than students' entire notebooks.

The intention is for students to be able to use their reference charts at any time, including during assessments. The goal is to learn to apply statements precisely, not to memorize. Some teachers ask students to make a tally mark each time they use a statement in the chart to justify a response. This allows students to see which are the most powerful statements and teachers to see how students are using their charts. Including the date will help students to know if they missed a row when they were absent or to locate a statement if they remember approximately how long ago they added it.

In addition to the blank reference chart, there is also a scaffolded version of the reference chart. The scaffolded version is intended to provide access for students with disabilities (language based, low vision, motor challenges) and English learners. In this version, students are provided with sentence frames for the "statement" column. The diagrams are also partially provided so students can focus on annotating key information. There is a teacher version of the chart in which the words needed to fill in the blanks and the missing annotations are highlighted.

Notation

Within student-facing text, these materials use words rather than symbols to allow students to focus on content instead of translating the meanings of symbols while reading. To increase exposure to different notation, images with information that is given by tick marks or arrows include a caption with the symbolic notation (like $\overline{AB} \cong \overline{CD}$). Teachers are encouraged to use the symbolic notation when recording student responses, since that is an appropriate use of shorthand.

Section A: Cross-Sections, Scaling, and Area

- Lesson 1: Solids
- Lesson 2: Solids of Rotation
- Lesson 3: Slicing Solids
- Lesson 4: Creating Cross-Sections by Dilating
- Lesson 5: Scaling and Area
- Lesson 6: Scaling and Unscaling

Section B: Scaling Solids

- Lesson 7: Scaling Solids
- Lesson 8: The Root of the Problem
- Lesson 9: Speaking of Scaling

Section C: Prism and Cylinder Volumes

- Lesson 10: Volume
- Lesson 11: Cylinder Volumes
- Lesson 12: Cross-Sections and Volume
- Lesson 13: Prisms Practice

Section D: Understanding Pyramid Volumes

- Lesson 14: Prisms and Pyramids
- Lesson 15: Building a Volume Formula for a Pyramid
- Lesson 16: Working with Pyramids
- Lesson 17: Putting All the Solids Together

Section E: Let's Put It to Work

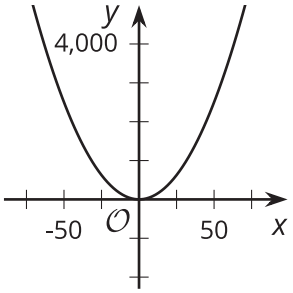
- Lesson 18: Surface Area and Volume
- Lesson 19: Volume and Density
- Lesson 20: Volume and Graphing

Unit 2: Polynomial Functions

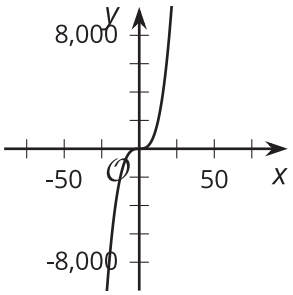
This unit extends students' previous work with linear and quadratic functions as they investigate polynomials of higher degree. Students rewrite polynomials in different forms, recognizing the benefits of the various forms for their ability to reveal the structure of key features of their graphs.

The unit begins with an introduction to two situations that can be modeled by a polynomial function. Students build their understanding of what polynomials are and what their graphs can look like. Certain aspects, such as end behavior, will be important in a later unit when students explore the end behavior of rational functions.

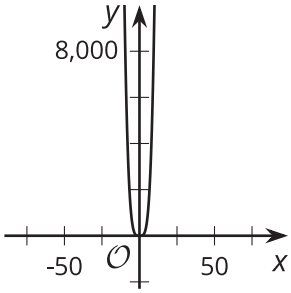
$y = x^2$



$y = x^3$



$y = x^4$



Focusing on functions expressed in factored form and their graphs, students connect that a factor of $(x - a)$ means a is a zero of the function and $(a, 0)$ is a horizontal intercept. The effect of the degree and leading coefficient on end behavior is established along with the effect of multiplicity on the shape of the graph near zeros of the function. Taking in all of these features, students learn to sketch polynomial functions expressed as a product of linear factors.

In a previous course, students used the distributive property to multiply factors, and also factored quadratics. Opportunities to review these skills and apply them to polynomials of higher degree are embedded throughout the unit and in practice problems. This prepares students for the final section where students divide a polynomial written in standard form by a suspected factor. From there, the connection between division and multiplication equations is used to establish the Remainder Theorem. This allows the conclusion that if a polynomial has a zero at $x = a$, then it must also have $(x - a)$ as a factor.

Section A: What Is a Polynomial?

- Lesson 1: Let's Make a Box
- Lesson 2: Funding the Future
- Lesson 3: Introducing Polynomials
- Lesson 4: Combining Polynomials

Section B: Working with Polynomials

- Lesson 5: Connecting Factors and Zeros
- Lesson 6: Different Forms
- Lesson 7: Using Factors and Zeros

Section C: Graphs of Polynomials

- Lesson 8: End Behavior (Part 1)
- Lesson 9: End Behavior (Part 2)
- Lesson 10: Multiplicity
- Lesson 11: Finding Intersections

Section D: Polynomial Division

- Lesson 12: Polynomial Division (Part 1)
- Lesson 13: Polynomial Division (Part 2)
- Lesson 14: What Do You Know about Polynomials?
- Lesson 15: The Remainder Theorem

Unit 3: Rationals, Radicals, and Identities

Building on the "Polynomial Functions" unit, in this unit, students transition to working with rational functions, rational equations, and identities.

The unit begins with an introduction to rational functions as students consider situations they can model, such as when minimizing surface area or determining average costs. Students examine the asymptotic behavior of their graphs, which relates to the structure of the equations. Students continue to build on structure as they use polynomial division to rewrite rational expressions for the purpose of identifying the end behavior of the function. Students then focus on solving rational equations and making sense of how some methods can lead to possible solutions that are in fact not solutions (so-called "extraneous solutions").

In the final section, students study identities. Students sharpen their skills manipulating expressions while proving, or disproving, that two expressions are equivalent. The unit concludes with a look at geometric sequences. Using a polynomial identity proved at the start of the section, students derive the formula for the sum of a finite geometric series before using the formula to solve problems.

Section A: Rational Functions

- Lesson 1: Minimizing Surface Area
- Lesson 2: Graphs of Rational Functions (Part 1)
- Lesson 3: Graphs of Rational Functions (Part 2)
- Lesson 4: End Behavior of Rational Functions

Section B: Rational Equations

- Lesson 5: Rational Equations (Part 1)
- Lesson 6: Rational Equations (Part 2)
- Lesson 7: Solving Rational Equations

Section C: Solving Equations with Square and Cube Roots

- Lesson 8: Inequivalent Equations?
- Lesson 9: Cubes and Cube Roots
- Lesson 10: Solving Radical Equations

Section D: Identities

- Lesson 11: Polynomial Identities (Part 1)
- Lesson 12: Polynomial Identities (Part 2)
- Lesson 13: A Radical Identity
- Lesson 14: Summing Up
- Lesson 15: Using the Sum

Section E: Let's Put It to Work

- Lesson 16: Drawing Proportional Circles

Unit 4: Exponential Functions and Equations

In this unit, students explore exponential and logarithmic functions. The unit begins with students recalling geometric sequences and drawing a connection to the growth or decay of values by a constant growth factor. This leads to expressing exponential relationships using functions of the form $f(x) = a \cdot b^x$, where a is the value of the function when $x = 0$ and b is the growth factor.

Students use different rational inputs, including negative values and values between integers, to better understand exponential functions and their meaning in various contexts. This includes an exploration of growth factors over intervals of different lengths. For example, the same population growth can be described using a growth factor per decade or a different growth factor per year.

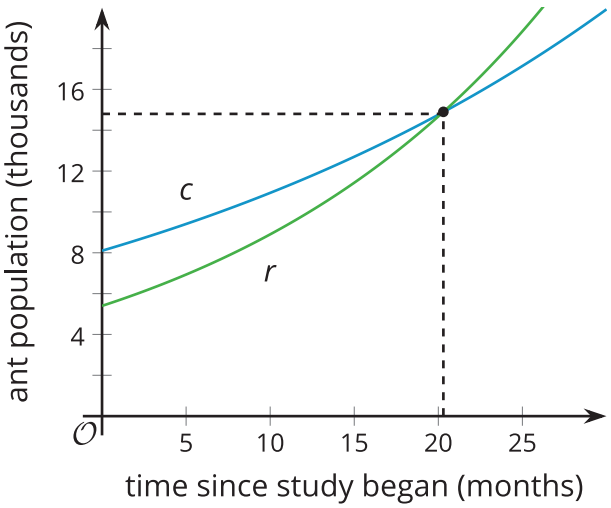
The exploration of variables used as an exponent leads to a need to solve equations for the variable and to the introduction of logarithms. Students are presented with traditional logarithm patterns and asked to find patterns and relationships, which leads them to discover that logarithms represent a way to rewrite exponential equations. That is, $a^y = x$ can be rewritten as $\log_a(x) = y$.

The constant e is introduced and students compare functions of the form $f(t) = P \cdot e^{rt}$ with functions of the form $g(t) = P \cdot (1 + r)^t$.

Then students explore logarithm rules including the product, quotient, and power rules as well as the change of base rule. The rules continue to reinforce that $\log_{10}(x) = \log(x)$ and $\log_e(x) = \ln(x)$, and the rules provide a way to use technology and the change of base rule to approximate other logarithms .

Finally, students solve exponential and logarithmic equations using graphical and algebraic methods and then interpret the solutions in context. The logarithms in this unit are primarily focused on the bases 10, 2, and e, although other positive bases are mentioned.

Note that, throughout the unit, the bases for both exponential expressions and logarithms are assumed to be positive and not equal to 1.



Section A: Growing and Shrinking

- Lesson 1: Growing and Shrinking
- Lesson 2: Representations of Growth and Decay
- Lesson 3: Understanding Rational Inputs
- Lesson 4: Representing Functions at Rational Inputs
- Lesson 5: Changes Over Rational Intervals

- Lesson 6: Writing Equations for Exponential Functions
- Lesson 7: Interpreting and Using Exponential Functions

Section B: Missing Exponents

- Lesson 8: Unknown Exponents
- Lesson 9: What Is a Logarithm?
- Lesson 10: Interpreting and Writing Logarithmic Equations
- Lesson 11: Evaluating Logarithmic Expressions

Section C: The Constant e

- Lesson 12: The Number e
- Lesson 13: Exponential Functions with Base e
- Lesson 14: Solving Exponential Equations

Section D: Logarithm Rules

- Lesson 15: Logarithm Product Rule
- Lesson 16: Logarithm Quotient Rule
- Lesson 17: Logarithm Power Rule
- Lesson 18: Logarithm Change of Base Rule
- Lesson 19: Using Logarithm Rules

Section E: Logarithmic Functions and Graphs

- Lesson 20: Using Graphs and Logarithms to Solve Problems (Part 1)
- Lesson 21: Using Graphs and Logarithms to Solve Problems (Part 2)
- Lesson 22: Logarithmic Functions

Section F: Let's Put It to Work

- Lesson 23: Applications of Logarithmic Functions

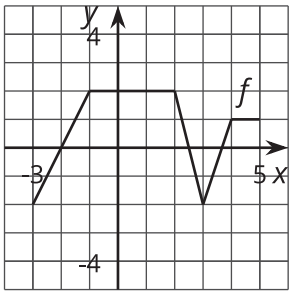
Unit 5: Transformations of Functions

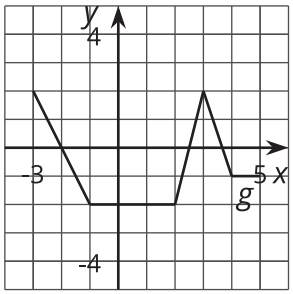
In this unit, students consider functions as a whole and understand how they can be transformed to fit the needs of a situation, which is an aspect of modeling with mathematics. Prior to this unit, students have worked with a variety of function types, such as polynomial, radical, and exponential. Students will build on this work in a later unit to transform periodic functions. By saving the introduction of trigonometric functions until after a study of transformations, students have the opportunity to revisit transformations from a new perspective, which reinforces the idea that all functions, even periodic ones, behave the same way with respect to translations, reflections, and scale factors.

The unit begins with students informally describing transformations of graphs, eliciting their prior knowledge, and establishing language that will be refined throughout the unit. Students begin their investigation with vertical and horizontal translations and reflections across the axes. This leads to both algebraic and geometric descriptions of a function as even, odd, or neither.

Students continue to deepen their understanding by exploring the effects of multiplying the input and output of a function by a scale factor. Students then apply their understanding of translations, reflections, and scaling to graphs and

equations of many function types. The unit ends with students applying transformations to different functions to model a real-world data set.





Section A: Translations, Reflections, and Symmetry

- Lesson 1: Matching Up to Data
- Lesson 2: Moving Functions
- Lesson 3: More Movement
- Lesson 4: Reflecting Functions
- Lesson 5: Some Functions Have Symmetry
- Lesson 6: Symmetry in Equations
- Lesson 7: Expressing Transformations of Functions Algebraically

Section B: Scaling Outputs and Inputs

- Lesson 8: Scaling the Outputs
- Lesson 9: Scaling the Inputs
- Lesson 10: Combining Functions

Section C: Transformations of Functions

- Lesson 11: Transforming from an Original Function
- Lesson 12: Transformation Effects
- Lesson 13: Transforming Parabolas
- Lesson 14: Transforming Circles

Section D: Let's Put It to Work

- Lesson 15: Making a Model for Data

Unit 6: Trigonometric Functions

In this unit, students are introduced to trigonometric functions. This unit builds directly on the work of a previous unit by having students apply their knowledge of transformations to trigonometric functions and use these functions to model periodic situations.

The unit begins with a deep study of the unit circle, as students study circular motion within familiar contexts. Students then use the Pythagorean Theorem and right-triangle trigonometry to determine the coordinates of points on a circle. They use the similarity of circles and right triangles to generalize to the unit circle, focusing on the important fact that when the hypotenuse has unit length, the length of the legs can be expressed with cosine and sine.

Then students transition to thinking about cosine and sine as functions. They use the unit circle to graph cosine and sine, then expand the domain to all real numbers as they learn the meaning of radian angles greater than 2π and less than 0. Students also reason about the input values where the tangent function does not exist and how the output values repeat at regular intervals, leading to the tangent function's periodic nature.

Next, students apply their previous work with transformations of graphs to trigonometric functions as they identify important features of periodic functions—including midline, amplitude, and period. They apply the work of this unit by modeling periodic or approximately periodic situations both algebraically and graphically.

Students create their own unit circle display in the unit. This display is meant to be a reference tool for students throughout the unit as they transition from a right-triangle-focused understanding of trigonometry to seeing cosine, sine, and tangent as functions with their own inputs and outputs. Students should have access to a unit circle display throughout the unit unless otherwise noted.

Section A: The Unit Circle

- Lesson 1: Moving in Circles
- Lesson 2: Revisiting Right Triangles
- Lesson 3: The Unit Circle (Part 1)
- Lesson 4: The Unit Circle (Part 2)
- Lesson 5: The Pythagorean Identity (Part 1)
- Lesson 6: The Pythagorean Identity (Part 2)
- Lesson 7: Finding Unknown Coordinates on a Circle

Section B: Periodic Functions

- Lesson 8: Rising and Falling
- Lesson 9: Introduction to Trigonometric Functions
- Lesson 10: Beyond 2π
- Lesson 11: Extending the Domain of Trigonometric Functions
- Lesson 12: Tangent
- Lesson 13: Some New Ratios

Section C: Trigonometry Transformations

- Lesson 14: Amplitude and Midline
- Lesson 15: Transforming Trigonometric Functions
- Lesson 16: Features of Trigonometric Graphs (Part 1)
- Lesson 17: Features of Trigonometric Graphs (Part 2)
- Lesson 18: Comparing Transformations
- Lesson 19: Modeling Circular Motion

Section D: Let's Put It to Work

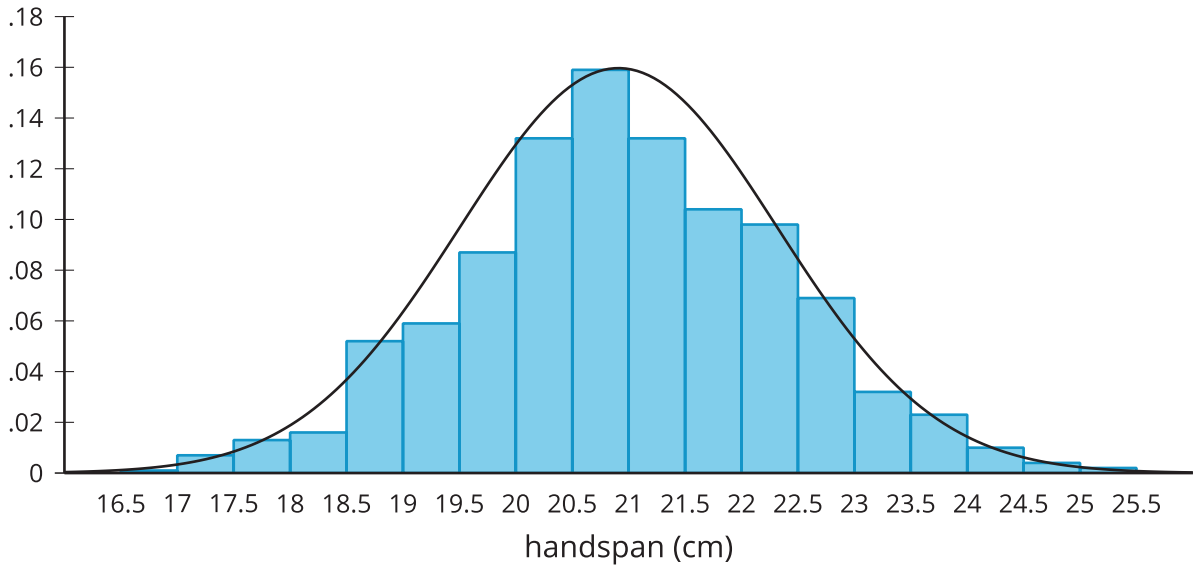
- Lesson 20: Beyond Circles

Unit 7: Statistical Inferences

This unit focuses on some important uses of randomization in statistics. Initially, students consider three types of study (experimental, observational, and survey) as ways of collecting data. In each form of study, random selection of participants and assignment into any subgroups is important for being able to generalize findings to a larger population. Students notice that an estimation of a proportion or mean of a population from sample data comes with some variability because different samples can result in different estimates. This can be expressed by including a margin of error along with any point estimates given.

Next, students examine the normal distribution as a common model for bell-shaped distributions. With technology, students can use a normal distribution model to approximate the proportion of data within certain intervals. This provides a way to quantify the confidence students should have in their estimates of population proportions or means. It also provides a way to check whether data from an experimental study has enough evidence to conclude that a difference in means between a control and treatment group is due to the treatment.

The unit concludes with an experimental study that students can do together, from study design to data collection and analysis. Then students draw a conclusion about the experiment based on their analysis.



Section A: Study Types

- Lesson 1: Being Skeptical
- Lesson 2: Study Types
- Lesson 3: Randomness in Groups

Section B: Distributions

- Lesson 4: Describing Distributions
- Lesson 5: Normal Distributions
- Lesson 6: Areas in Histograms
- Lesson 7: Areas under a Normal Curve

Section C: Not All Samples Are the Same

- Lesson 8: Not Always Ideal

- Lesson 9: Variability in Samples
- Lesson 10: Estimating Proportions from Samples
- Lesson 11: Estimating a Population Mean

Section D: Analyzing Experimental Data

- Lesson 12: Experimenting
- Lesson 13: Using Normal Distributions for Experiment Analysis
- Lesson 14: Questioning Experimenting

Section E: Let's Put It to Work

- Lesson 15: Heart Rates

Pacing Guide

Number of days includes assessments. Upper bound of range includes optional lessons. Does not include time for modeling prompts.

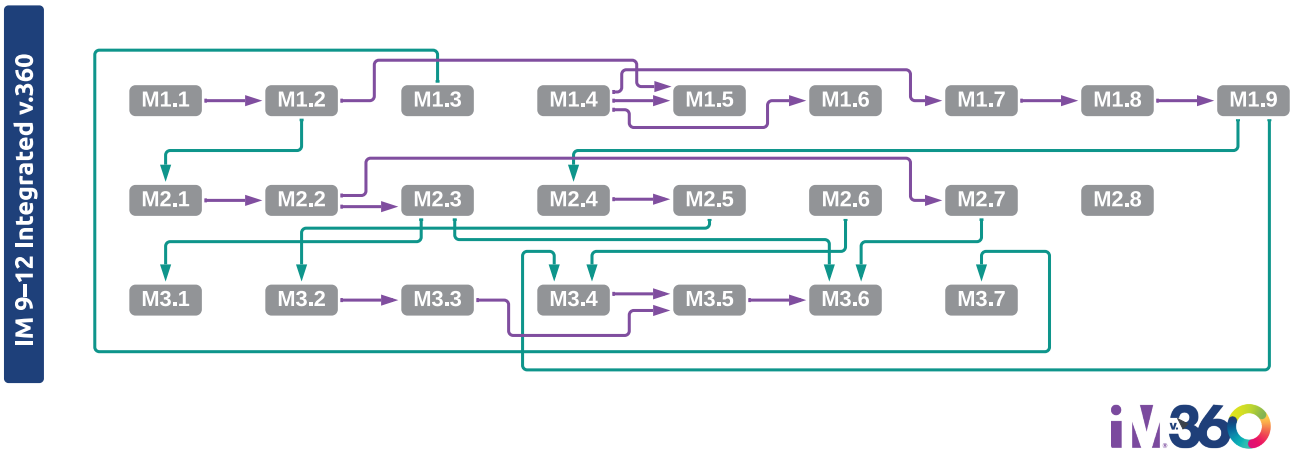
	Math 1	Math 2	Math 3
week 1	Unit 1 (MA) Constructions and Rigid Transformations 20–22 days Optional Lessons: 8, 18	Unit 1 Convincing Arguments 16 days Optional lessons: none	Unit 1 Solid Geometry 21–22 days Optional Lessons: 10
week 2			
week 3			
week 4		Unit 2 Similarity 17–20 days Optional Lessons: 2, 10, 14	
week 5	Unit 2 Congruence 12–13 days Optional Lesson: 11		Unit 2 Polynomial Functions 17 days Optional Lessons: none
week 6			
week 7			
week 8	Unit 3 One-Variable Statistics 13–18 days Optional Lessons: 2, 5, 6, 7, 8	Unit 3 Right Triangle Trigonometry 14 days Optional Lessons: none	
week 9			Unit 3 Rationals, Radicals, and Identities 16–18 days Optional Lessons: 10, 16
week 10			
week 11	Unit 4 Linear Equations and Systems 16–21 days Optional Lessons: 2, 4, 5, 18, 19	Unit 4 (MA) Introduction to Quadratic Functions 19–22 days Optional Lesson: 13, 14, 16	
week 12			
week 13			Unit 4 (MA) Exponential Functions and Equations 21–24 days Optional Lessons: 2, 7, 23
week 14			
week 15	Unit 5 Coordinate Geometry 12–13 days Optional Lessons: 10		
week 16		Unit 5 (MA) Quadratic Equations 26–27 days Optional Lessons: 18	
week 17			
week 18	Unit 6 Two-Variable Statistics 11–12 days Optional Lesson: 10		Unit 5 Transformations of Functions 17 days Optional Lessons: none
week 19			
week 20			
week 21	Unit 7 Linear Inequalities and Systems 11 days Optional Lessons: none	Unit 6 Complex Numbers 13–17 days Optional: 1, 2, 11, 15	
week 22			
week 23			Unit 6 (MA) Trigonometric Functions 23 days Optional Lessons: none
week 24	Unit 8 (MA) Functions 23 days Optional Lessons: none		
week 25			
week 26		Unit 7 Circles 20 days Optional Lessons: none	
week 27			Unit 7 (MA) Statistical Inferences 17–18 days Optional Lesson: 4
week 28	Unit 9 (MA) Introduction to Exponential Functions 22–24 days Optional Lesson: 13, 14		
week 29			
week 30		Unit 8 Conditional Probability 11 days Optional Lessons: 1, 11	
week 31			
week 32			

(MA) = Unit has Mid-Unit Assessment

Total number of days = Lessons + Assessments – Optional Lessons

Math 1 = 140, Math 2 = 136, Math 3 = 132

Dependency Chart



In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. For example, there is an arrow from M1.8 to M1.9 because when exponential functions are introduced, function notation is used, assuming that students are already familiar with the notation.

The following chart shows unit dependencies between 6–8 and Integrated Math.

