Unit 5, Lesson 11

Evaluating Logarithmic Expressions

Let's find some logs!

11.1 Math Talk: Finding Values

Evaluate mentally.

- $\log 10$
- $\log 10{,}000$
- $\log 0.1$
- $\log \frac{1}{1{,}000}$

11.2 Log War!

Have you played the game of war with a deck of playing cards?

Your teacher will give you and your partner a set of special cards.

- Shuffle and deal the cards evenly so that each player has half of the cards in a face-down stack.
- Each player turns one card face up. The card with the greater value wins the round, and its player captures both cards and sets them aside.
- If you and your partner disagree about the value of a card, discuss until you reach an agreement.
- If there is a tie, each player turns another card face up. The player whose card has the greater value captures all cards (including the cards that tie).
- Play until all the cards are turned up. The player with the most cards wins.

Let the logarithm war begin!

Are you ready for more?

Mai uses the fact that $\sqrt[3]{2}$ is close to 1.25 and that $2^3 = 8$ to make the estimate that $\log_2(10) \approx 3\frac{1}{3}$. Explain Mai's estimate. How exact is the estimate?

11.3 Finding Logarithms with a Calculator

1. To solve the equation $10^m = 19$, Tyler writes the equation in the logarithmic form: $m = \log_{10} 19$. He then presses the "log" button on the calculator, enters the number "19," and writes down an approximation of 1.279. Priya follows the same steps on her calculator and writes down 1.27875.
   a. Experiment with your calculator until you understand how to evaluate $\log_{10} 19$. What value do you see on the calculator?

   b. Discuss with a partner: Why might $\log_{10} 19$ be expressed in different ways?
   c. How could Priya use the original exponential equation to check the value from her calculator?

2. Express the solution to each equation using a logarithm. Next, use a calculator to find the approximate value of the solution.
   a. $10^m = 24$

   b. $10^n = 750$

3. Estimate the value of each expression without a calculator. Explain to a partner how you made your estimate. Next, check your estimate with a calculator.
   a. $\log 90$

   b. $\log 1{,}005$

   c. $\log 9$

Lesson 11 Summary

Sometimes it's possible to find an exact value for a logarithm. For example $\log_2 0.125 = -3$ because $0.125 = \frac{1}{8}$, and $2^{-3} = \frac{1}{8}$. Similarly, $\log_5 625 = 4$ because $5^4 = 625$.

Often it is not possible to find an exact value, but using number sense allows us to get a reasonable estimate. Let's say we want to find x that makes $10^x = 980$ true.

We can first express x as $\log_{10} 980$. Then, because 980 is between 10^2 and 10^3, we know that $\log_{10} 980$ must be between 2 and 3.

But where does $\log_{10} 980$ lie between 2 and 3? Because 980 is much closer to 1,000 than it is to 100, $\log_{10} 980$ is likely a lot closer to 3 than it is to 2. This means that 2.9 would be a better estimate than 2.1 would be.

We can use a calculator to verify our estimate and find that $\log_{10} 980$ is about 2.99.