Other Fractional Exponents

Let's look at powers of roots.

10.1 Breaking Up Fractions

1. Write these products as a single fraction.
   a. $3 \cdot \frac{1}{5}$
   b. $10 \cdot \frac{1}{9}$
   c. $\frac{1}{8} \cdot 6$
2. Write these fractions as the product of a whole number and a fraction with a numerator of 1.
   a. $\frac{2}{3}$
   b. $\frac{5}{4}$
   c. $1\frac{3}{5}$

10.2 Other Rational Exponents

Rewrite each expression using one or more roots (like $\sqrt{2}$ or $\sqrt[3]{2}$) and whole-number exponents.

1. $\left(11^{\frac{1}{2}}\right)^3$
2. $\left(b^3\right)^{\frac{1}{4}}$
3. $3^{\frac{2}{5}}$
4. $\left(a^2 \cdot b^4\right)^{\frac{1}{5}}$
5. $\left(x^3 \cdot b^9\right)^{\frac{2}{3}}$
6. $\left(y^{\frac{1}{2}} z^{\frac{3}{4}}\right)^8$

Are you ready for more?

From the commutative property of multiplication, $a \cdot \frac{1}{b} = \frac{1}{b} \cdot a$.

1. Use exponents to show that $\left(\sqrt[b]{x}\right)^a = \sqrt[b]{x^a}$.

2. Use technology and numbers for x, a, and b to check that this is true, or describe patterns of numbers for when it is not true.

10.3 Half Days

Scientists are growing bacteria for an experiment. From past experiments, scientists estimate that the bacteria will grow in this environment so that the population t days after the experiment is started is $300 \cdot 2^t$.

1. What is the value of the expression when $t = 0$? What does it mean in this situation?

2. What is the value of the expression when $t = 1$? What does it mean in this situation?

3. What is the value of the expression when $t = \frac{1}{2}$? What does it mean in this situation?

Lesson 10 Summary

We can extend our connection between expressions with fractional exponents and expressions with roots to $a^{\frac{b}{c}} = \left(\sqrt[c]{a}\right)^b$.

This means that we can interpret an expression like $3^{\frac{2}{5}}$ as $\left(\sqrt[5]{3}\right)^2$ or as $\sqrt[5]{3^2} = \sqrt[5]{9}$.

Using the exponent rule can also be useful for writing roots in a simpler way. For example, it is true that $\sqrt[4]{3^{10}} = \sqrt{3^5}$ because the first expression can be written as $\sqrt[4]{3^{10}} = 3^{\frac{10}{4}}$, which is equivalent to $3^{\frac{5}{2}} = \sqrt{3^5}$.

The rule can also be helpful when interpreting certain situations. For example, let's fill a chess board with rice by putting 1 grain of rice on the top left square, then doubling the amount of rice on each square as we go across the board. Will there ever be 1 million grains of rice on a square? If so, when?

We could write a function to represent the number of grains of rice on a square as $R(s) = 2^s$, where s represents the number of times the amount of rice on a square is doubled, so that there is 1 grain of rice to start, 2 grains after it has been doubled once, 4 grains after doubling twice, and so on. By graphing $R(s)$ and $y = 1{,}000{,}000$ we could figure out how many times it has been doubled to have 1 million grains of rice on a square. It turns out that the graphs intersect when $s = 19.932$. This means that when we double the rice on a square 20 times it will have more than 1 million grains of rice on it (and we are doubling the rice 43 more times for the last square of the board)!