Logarithm Quotient Rule

Let's combine logarithms that are subtracted.

16.1 A Pattern in Logarithm Differences

For each difference, find the value of the difference by finding the value of each logarithm and then subtracting the values. Then complete the given logarithm so that it has the same value as the difference.

The first one is done for you. Discuss with your partner why it is true.

1. $\log_5(625) - \log_5(5) = \underline{3}$

   $\log_5(625) - \log_5(5) = \log_5(\underline{125})$

2. $\log_6(36) - \log_6(6) =$ ________

   $\log_6(36) - \log_6(6) = \log_6($________$)$

3. $\log_3(9) - \log_3(1) =$ ________

   $\log_3(9) - \log_3(1) = \log_3($________$)$

4. $\log_2(16) - \log_2(32) =$ ________

   $\log_2(16) - \log_2(32) = \log_2($________$)$

5. $\log(10{,}000) - \log(100) =$ ________

   $\log(10{,}000) - \log(100) = \log($________$)$

16.2 Making a Conjecture about Logarithm Differences

1. Use the pattern you noticed about differences of logarithms with the same base to write a conjecture.

$$\log_b(U) - \log_b(V) = \log_b\left(\underline{\hspace{3cm}}\right)$$

2. Assume the conjecture is true. Rewrite each expression as a single logarithm, then find its value.

a. $\log_6(72) - \log_6(2)$

b. $\log(12) - \log(0.12)$

c. $\log_{12}(2) - \log_{12}(24)$

3. If $\log(3) = 0.4771$, $\log(4) = 0.6021$, and $\log(120) = 2.0792$, find the values of each logarithm. Explain or show your reasoning.

   a. $\log\left(\frac{4}{3}\right)$

   b. $\log(30)$

   c. $\log(0.03)$

16.3 Proving the Conjecture about Logarithm Differences

Let's work through some steps of a proof for your conjecture.

Start with two equations: $b^x = U$ $\quad$ $b^y = V$

1. Rewrite both of these equations as logarithms, and circle your answers to use later. $\quad \log(\quad) \quad \log(\quad)$
2. Divide the left sides of the original equations, and set the quotient equal to the quotient of the right sides of the original equations. $\quad \underline{\qquad\qquad} = \frac{U}{V}$
3. Combine the exponents on the left side of the equation so that it is written with a single base. $\quad \underline{\qquad\qquad} = \frac{U}{V}$
4. Rewrite the last equation as a logarithm. $\quad \log(\quad)$
5. Use your circled equations to replace any x and y in that equation with equivalent logarithms.

Are you ready for more?

Before electronic calculators were readily available, people used this rule to divide large numbers using logarithm tables such as the ones earlier in this unit. To see an example, research how to use a slide rule.

Lesson 16 Summary

The **quotient rule** for logarithms allows us to combine a difference of logarithms with the same base into a single logarithm. The quotient rule states that

$$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$$

For example, $\log(15) - \log(5) = \log(3)$.

Thinking about logarithms in relation to exponents, this may make more sense. We learned in an earlier course that

$$\frac{a^x}{a^y} = a^{x-y}$$

By rewriting parts of that equation into their logarithm form, we can combine the pieces to prove the quotient rule.