Proving the Triangle Congruence Theorems
Sentence Frames for Proofs

Transformations:

- Translate _____ from _____ to _____.
- Rotate _____ using _____ as the center by angle _____.
- Rotate _____ using _____ as the center so that _____ coincides with _____.
- Reflect _____ across _____.
- Reflect _____ across the perpendicular bisector of _____.
- Segments _____ and _____ are the same length so they are congruent. Therefore, there is a rigid motion that takes _____ to _____. Apply that rigid motion to _____.

Justifications:

- We know the image of _____ is congruent to _____ because rigid motions preserve measure.
- Points _____ and _____ coincide after translating because we defined our translation that way!
- Since points _____ and _____ are the same distance along the same ray from _____ they have to be in the same place.
- Rays _____ and _____ coincide after rotating because we defined our rotation that way!
- The image of _____ must be on ray _____ since both _____ and _____ are on the same side of _____ and make the same angle with it at _____.
- Points _____ and _____ coincide because they are both at the intersection of the same lines, and 2 distinct lines can only intersect in 1 place.
- _____ is the perpendicular bisector of the segment connecting _____ and _____, because the perpendicular bisector is determined by 2 points that are both equidistant from the endpoints of a segment.

Conclusion statement:

- We have shown that a rigid motion takes _____ to _____, _____ to _____, and _____ to _____, therefore triangle _____ is congruent to triangle _____.

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Using the Triangle Congruence Theorems

More Proof Supports

Many proofs in Euclidean geometry that don't use transformations use congruent triangles: if you can find two triangles that you are SURE are congruent, you can prove that any corresponding parts of your triangles are congruent.

1. Can you find any triangles that are probably congruent? Suggestion: outline them in different colors or re-draw them separately on your paper.
2. If you can't find any triangles yet, is there a helpful auxiliary line you can draw?
    a. A line of symmetry?
    b. A segment connecting two points, such as the diagonal of a quadrilateral?
3. Label all of the things you know are congruent. This will help you decide how to prove two triangles are congruent.
    a. Do you know all three pairs of corresponding sides are congruent? Use SSS Congruence!
    b. Do you know two pairs of corresponding angles are congruent? Look to see if you can show the sides between the corresponding angles are congruent to use ASA Congruence!
    c. Do you know two pairs of corresponding sides are congruent? Look to see if you can show the angles between the corresponding sides are congruent to use SAS!
4. Seems like there's not enough information? Here are some things to check:
    a. Do the triangles share a side or an angle? Sides and angles are congruent to themselves!
    b. Are any of the sides radii of the same circle? All of the radii in the same circle are congruent.
    c. Are there parallel lines? Look for angles that must be congruent when formed by parallel lines, such as alternate interior angles.
    d. Are there vertical angles?
    e. Is there a quadrilateral with special properties?

You can use this template if you want:

Goal: Prove _____ is congruent to _____

I'm going to do this by proving Triangle _____ is congruent to triangle _____ by _____ Congruence Theorem.

Statement 1:

Reason 1:

Statement 2:

Reason 2:

Statement 3:

Reason 3:

Therefore, Triangle _____ is congruent to triangle _____ by _____ Congruence Theorem.

Since _____ and _____ are corresponding parts of congruent triangles, _____ and _____ must be congruent.

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