Unit 8, Lesson 11

Variability and MAD

Let's study distances between data points and the mean and see what they tell us.

11.1 Shooting Hoops (Part 1)

Elena, Jada, and Lin enjoy playing basketball during recess. Lately, they have been practicing free throws. They record the number of baskets they make out of 10 attempts. Here are their data sets for 12 school days.

Elena

2	2	2	2	4	5	5	6	8	9	9	9

Jada

2	4	5	4	6	6	4	7	3	4	8	7

Lin

3	6	6	4	5	5	3	5	4	6	6	7

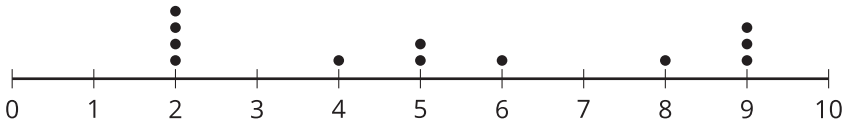
1. Calculate the mean number of baskets each player made, and compare the means. What do you notice?

2. What do the means tell us in this context?

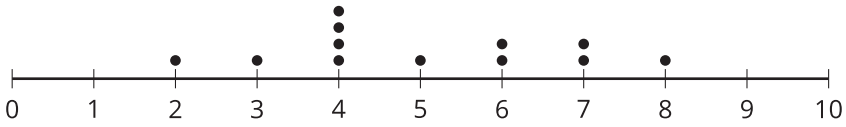
11.2 Shooting Hoops (Part 2)

Here are the dot plots showing the number of baskets that Elena, Jada, and Lin each made over 12 school days.

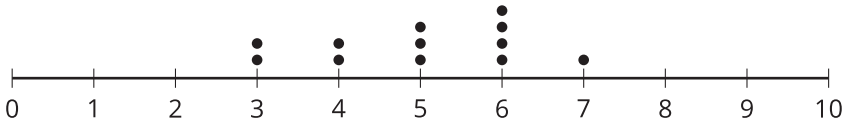
1. On each dot plot, mark the location of the mean with a triangle. Then, contrast the dot plot distributions. Write 2–3 sentences to describe the shape and spread of each distribution.



number of baskets Elena made



number of baskets Jada made



number of baskets Lin made

2. Discuss these questions with your group. Explain your reasoning.
   a. Would you say that all three students play equally well?
   b. Would you say that all three students play equally consistently?
   c. If you could choose one player to be on your basketball team based on their records, whom would you choose?

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11.3 Shooting Hoops (Part 3)

The tables show the number of baskets made by Jada and Lin in several games. Recall that the mean of Jada and Lin's data is 5.

1. Record the distance between the number of baskets Jada made in each game and the mean.

Jada	2	4	5	4	6	6	4	7	3	4	8	7
distance from 5												

Now find the average of the distances in the table. Show your reasoning, and round your answer to the nearest tenth.

This value is the mean absolute deviation (MAD) of Jada's data. Jada's MAD: _____

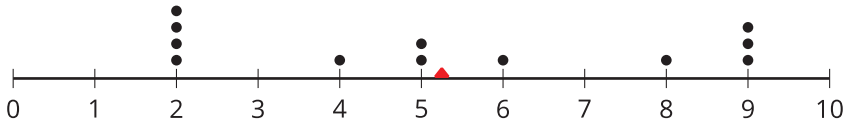
2. Find the mean absolute deviation of Lin's data. Round it to the nearest tenth.

Lin	3	6	6	4	5	5	3	5	4	6	6	7
distance from 5												

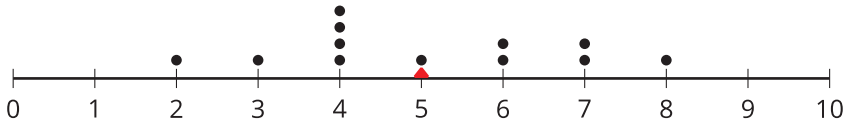
Lin's MAD: _____

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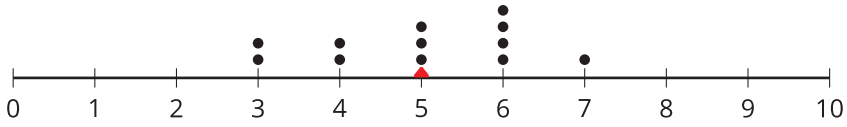
3. Elena's distribution has a MAD of about 2.5. Compare the MADs and dot plots of the three students' data. Do you see a relationship between each student's MAD and the distribution on her dot plot? Explain your reasoning.



number of baskets Elena made



number of baskets Jada made



number of baskets Lin made

Are you ready for more?

Invent another data set that also has a mean of 5 but has a MAD greater than 2. Remember, the values in the data set must be whole numbers from 0 to 10.

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11.4 Game of 22

Your teacher will give your group a deck of cards. Shuffle the cards, and put the deck face down on the playing surface.

- To play: Draw 3 cards and add up the values. An ace is a 1. A jack, queen, and king are each worth 10. Cards 2–10 are each worth their face value. If your sum is anything other than 22 (either above or below 22), say: "My sum deviated from 22 by ____ ," or "My sum was off from 22 by ____ ."
- To keep score: Record each sum and each distance from 22 in the table. After five rounds, calculate the average of the distances. The player with the lowest average distance from 22 wins the game.

player A	round 1	round 2	round 3	round 4	round 5
sum of cards					
distance from 22					

Average distance from 22: _____

player B	round 1	round 2	round 3	round 4	round 5
sum of cards					
distance from 22					

Average distance from 22: _____

player C	round 1	round 2	round 3	round 4	round 5
sum of cards					
distance from 22					

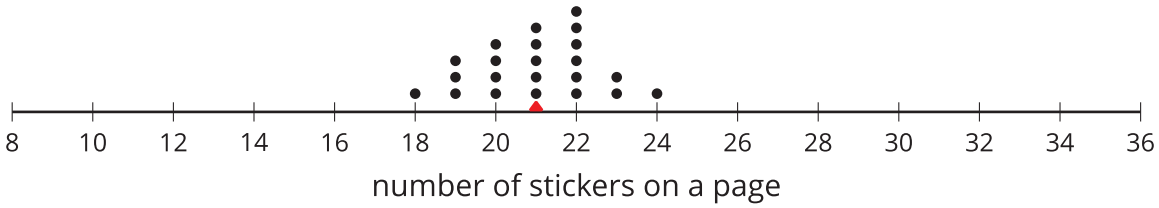
Average distance from 22: _____

Whose average distance from 22 is the smallest? Who won the game?

Lesson 11 Summary

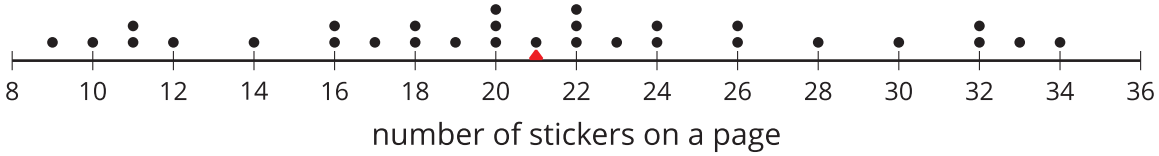
We use the mean of a data set as a "measure of center" of its distribution, but two data sets with the same mean could have very different distributions.

This dot plot shows the number of stickers on each page of a 22-page sticker book.



The mean number of stickers is 21. All the pages have within 3 stickers of the mean, and most of them are even closer. These pages are all fairly close in the number of stickers on them.

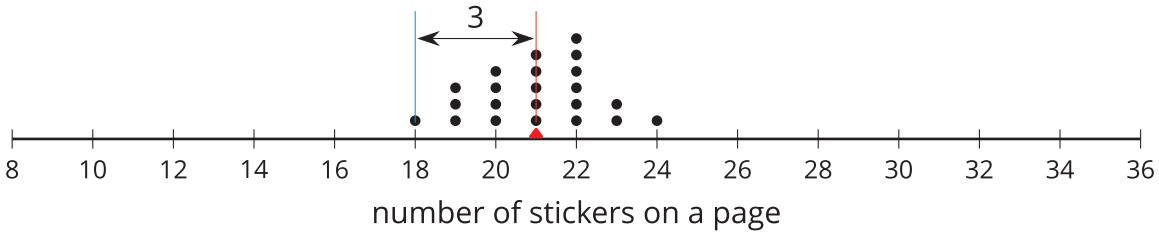
This dot plot shows the number of stickers on each page of another sticker book that has 30 pages.



In this sticker book, the mean number of stickers on each page is also 21, but some pages have less than half that number of stickers and others have more than one-and-a-half times as many. There is a lot more variability in the number of stickers.

There is a number that we can use to describe how far away, or how spread out, data points generally are from the mean. This *measure of spread* is called the **mean absolute deviation (MAD)**.

To find the MAD, we find the distance between each data value and the mean, and then calculate the mean of those distances. For instance, the point that represents 18 stickers is 3 units away from the mean of 21 stickers.



We can find the distance between each point and the mean of 21 stickers and then organize the distances into a table, as shown.

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number of stickers	18	19	19	19	20	20	20	20	21	21	21	21	21	22	22	22	22	22	22	23	23	24
distance from mean	3	2	2	2	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	2	2	3

The values in the first row of the table are the number of stickers on each page in the first book. Their mean, 21, is the mean number of stickers on a page.

The values in the second row of the table are the distances, or absolute deviation, between the values in the first row and 21. The mean of these distances is the MAD of the number of stickers on a page, about 1.2 stickers.

What can we learn from the averages of these distances once they are calculated?

- In the first book, the distances are all between 0 and 3. The MAD is 1.2 stickers, which tells us that the number of stickers are typically within 1.2 of the mean number, 21. We could say that a typical page has between 19.8 and 22.2 stickers.
- In the second book, the distances are all between 0 and 13. The MAD is 5.6 stickers, which tells us that the number of stickers are typically within 5.6 of the mean number, 21. We could say that a typical page has between 15.4 and 26.6 stickers.

The MAD is also called a *measure of the variability* of the distribution. In these examples, it is easy to see that a higher MAD suggests a distribution that is more spread out, showing more variability.