Unit 6, Lesson 15

Efficiently Solving Inequalities

Let's solve more complicated inequalities.

15.1 Lots of Negatives

Here is an inequality: $-x \geq -4$

1. Without testing any values, predict what the solutions to this inequality will look like on the number line.
   - Where will the starting (or boundary) point be?
   - Will the circle at that point be filled or unfilled?
   - Will the shading go to the right or to the left of that point?
2. Test each value to see whether it is a solution to the inequality $-x \geq -4$.
   a. 3
   b. -3
   c. 4
   d. -4
   e. 4.001
   f. -4.001
3. Graph all possible solutions to the inequality on the number line:



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15.2 Inequalities with Tables

1. Let's investigate the inequality $x - 3 > -2$.

x	-4	-3	-2	-1	0	1	2	3	4
$x - 3$	-7	-6		-4			-1		1

   a. Complete the table.

   b. For which values of x is it true that $x - 3 = -2$?

   c. For which values of x is it true that $x - 3 > -2$?

   d. Graph the solutions to $x - 3 > -2$ on the number line:



2. Here is an inequality: $2x < 6$.

   a. Predict which values of x will make the inequality $2x < 6$ true.

   b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$2x$									

   c. Graph the solutions to $2x < 6$ on the number line:



3. Here is an inequality: $-2x < 6$.

   a. Predict which values of x will make the inequality $-2x < 6$ true.

   b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$-2x$									

   c. Graph the solutions to $-2x < 6$ on the number line:



4. How are the solutions to $2x < 6$ different from the solutions to $-2x < 6$?

15.3 Which Side Shows the Solutions?

1. Let's investigate $-4x + 5 \geq 25$.

   a. Solve $-4x + 5 = 25$.

   b. Is $-4x + 5 \geq 25$ true when x is 0? What about when x is 7? What about when x is -7?

   c. Graph the solutions to $-4x + 5 \geq 25$ on the number line.



2. Let's investigate $\frac{x}{3} + 7 < 19$.

   a. Solve $\frac{x}{3} + 7 = 19$.

   b. Is $\frac{x}{3} + 7 < 19$ true when x is 0?

   c. Graph the solutions to $\frac{x}{3} + 7 < 19$ on the number line.



3. Solve the inequality $3(x - 4) > 16.5$, and graph the solutions on the number line.



4. Solve the inequality $-2\left(x + \frac{3}{2}\right) \leq 6$, and graph the solutions on the number line.



Are you ready for more?

Write at least three different inequalities whose solution is $x > -10$. Find one inequality with x on the left side that uses a $<$.

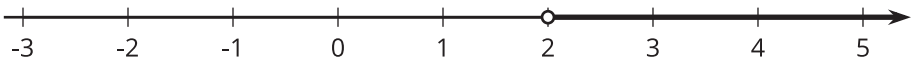
Lesson 15 Summary

Here is an inequality: $3(10 - 2x) < 18$. The solution set for this inequality is all the values that can be used in place of x to make the inequality true. Each solution is one value that makes the inequality true.

In order to solve this inequality, we can first solve the related equation $3(10 - 2x) = 18$ to get the solution $x = 2$. That means 2 is the boundary between values of x that make the inequality true and values that make the inequality false.

To solve the inequality, we can check numbers greater than 2 and less than 2 and see which ones make the inequality true.

Let's check a number that is greater than 2: $x = 5$. Replacing x with 5 in the inequality, we get $3(10 - 2 \cdot 5) < 18$ or just $0 < 18$. This is true, so $x = 5$ is a solution. This means that all values greater than 2 make the inequality true. We can represent the solutions as $x > 2$ and also represent the solutions on a number line:



Notice that 2 itself is not a solution because it's the value of x that makes $3(10 - 2x)$ *equal* to 18, and so it does not make $3(10 - 2x) < 18$ true.

For confirmation that we found the correct solution, we can also test a value that is less than 2. If we test $x = 0$, we get $3(10 - 2 \cdot 0) < 18$ or just $30 < 18$. This is false, so $x = 0$ and all values of x that are less than 2 are not solutions.