Unit 7 Family Support Materials

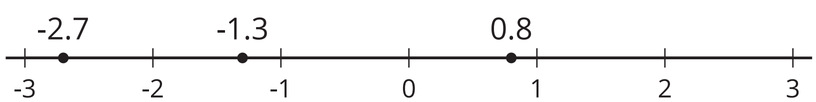
Rational Numbers

Section A: Negative Numbers and Absolute Value

This week, your student will work with signed numbers, or **positive** numbers and **negative numbers**. We often compare signed numbers when talking about temperatures. For example, -30 degrees Fahrenheit is colder than -10 degrees Fahrenheit. We say “-30 is less than -10” and write: $-30 < -10$.

We also use signed numbers when referring to elevation, or height relative to the sea level. An elevation of 2 feet (which means 2 feet above sea level) is higher than an elevation of -4 feet (which means 4 feet below sea level). We say, “2 is greater than -4” and write, "$2 > -4$."

We can plot positive and negative numbers on the number line. Numbers to the left are always less than numbers to the right.



We can see that -1.3 is less than 0.8 because -1.3 is to the left of 0.8, but -1.3 is greater than -2.7 because it is to the right of -2.7.

We can also talk about a number in terms of its **absolute value**, or its distance from 0 on the number line. Here are some examples:

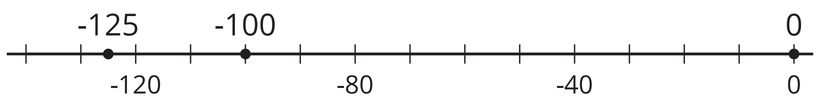
- 0.8 is 0.8 unit away from 0. To represent this statement, we can write, "$|0.8| = 0.8$."
- -2.7 is 2.7 units away from 0. To represent this statement, we can write, "$|-2.7| = 2.7$."
- The numbers -3 and 3 are both 3 units away from 0. We can write, "$|3| = 3$ and $|-3| = 3$."

Here is a task to try with your student:

1. A diver is at the surface of the ocean, getting ready to do a dive. What is the diver's elevation in relation to sea level?
2. The diver descends 100 feet to the top of a wrecked ship. What is the diver's elevation now?
3. The diver descends 25 feet more, toward the ocean floor. What is the absolute value of the diver's elevation now?
4. Plot each of these three elevations as a point on a number line. Label each point with its numeric value.

Solution:

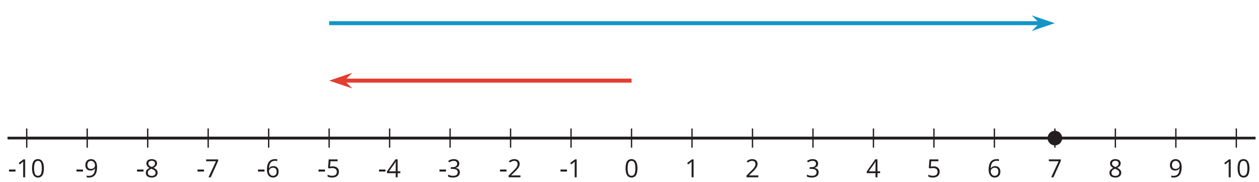
1. 0, because sea level is 0 feet above or below sea level
2. -100, because the diver is 100 feet *below* sea level
3. The new elevation is -125 feet, or 125 feet *below* sea level, so its absolute value is 125 feet.
4. A number line with 0, -100, and -125 marked, as shown:



Section B: Adding and Subtracting Rational Numbers

This week your student will be adding and subtracting with **negative numbers**. We can represent this on a number line using arrows. The arrow for a **positive number** points right, and the arrow for a negative number points left. We add numbers by putting the tail of the second arrow at the tip of the first arrow.

For example, here is a number line that shows $-5 + 12 = 7$.



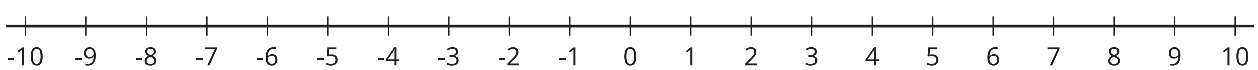
The first number is represented by an arrow that starts at 0 and points 5 units to the left. The next number is represented by an arrow that starts directly above the tip of the first arrow and points 12 units to the right. The answer is 7 because the tip of the second arrow ends above the 7 on the number line.

In elementary school, students learned that every addition equation has two related subtraction equations. For example, if we know $3 + 5 = 8$, then we also know $8 - 5 = 3$ and $8 - 3 = 5$.

The same thing works when there are negative numbers in the equation. From the previous example, $-5 + 12 = 7$, we also know $7 - 12 = -5$ and $7 - -5 = 12$.

Here is a task to try with your student:

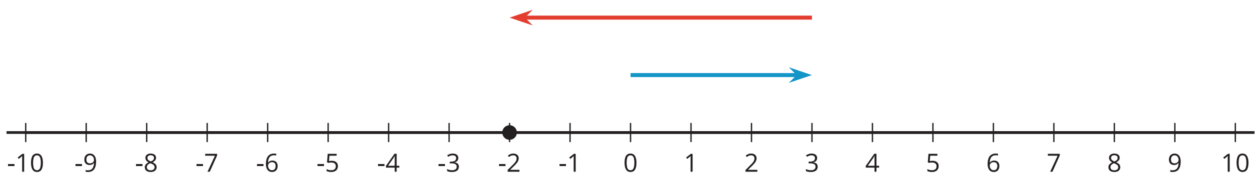
1. Use the number line to show $3 + -5$.



2. What does your answer tell you about the value of:
   a. $-2 - 3$?
   b. $-2 - -5$?

Solution:

1. The first arrow starts at 0 and points 3 units to the right. The next arrow starts at the tip of the first arrow and points 5 units to the left. This arrow ends above the -2, so $3 + -5 = -2$.

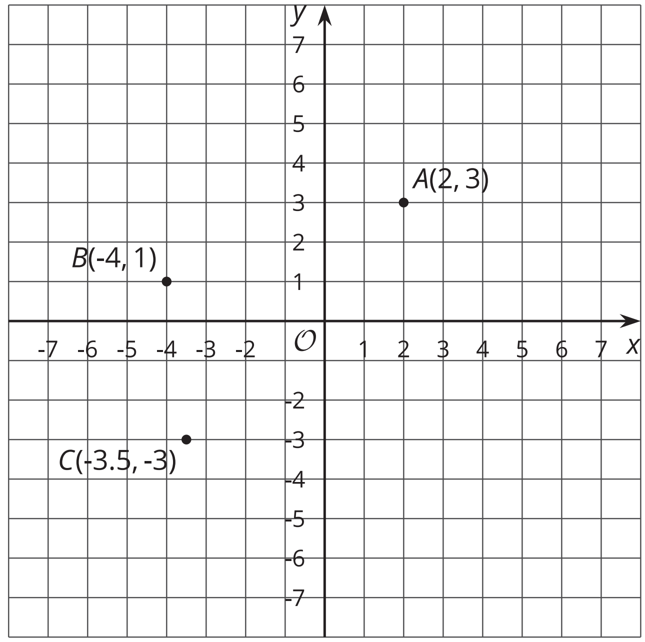


2. From the addition equation $3 + -5 = -2$, we get the related subtraction equations:
   a. $-2 - 3 = -5$
   b. $-2 - -5 = 3$

Section C: The Coordinate Plane

This week, your student will plot and interpret points in the coordinate plane.

In earlier grades, they plotted points where both coordinates were positive, such as point A in the figure. They will now plot points that have one positive coordinate and one negative coordinate, such as point B, and points that have two negative coordinates, such as point C.

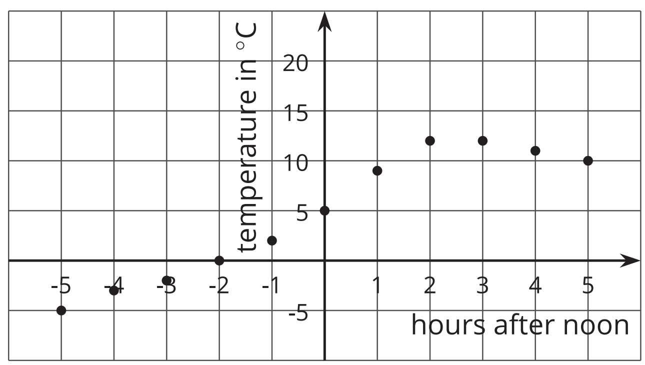


To find the distance between two points that are on the same horizontal or vertical line, we can simply count the grid units between them.

For example, if we plot the point $(2, -4)$ on this grid, we can tell that the point will be 7 units away from point $A(2, 3)$.

Points in a coordinate plane can also represent situations involving positive and negative numbers.

For instance, the points in this coordinate plane show the temperature in degrees Celsius every hour before and after noon on a winter day. Times before noon are negative and times after noon are positive.



The point $(5, 10)$ tells us that 5 hours after noon, or at 5:00 p.m, the temperature was 10 degrees Celsius.

Here is a task to try with your student:

In the graph about temperatures before and after noon:

1. What was the temperature at 7 a.m.? Which point represents this information?
2. For which recorded times was it colder than 5 degrees Celsius?

Solution:

1. It was -5 degrees Celsius at 7:00 a.m. The point $(-5, -5)$ represents this information.
2. It was colder than 5 degrees Celsius for all of the recorded times before noon.

Section D: Multiplying and Dividing Rational Numbers

This week your student will be multiplying and dividing with negative numbers. The rules for multiplying positive and negative numbers are designed to make sure that addition and multiplication work the same way they always have.

For example, in elementary school students learned to think of “4 times 3” as 4 groups of 3, like $4 \cdot 3 = 3 + 3 + 3 + 3 = 12$. We can think of “4 times -3” the same way: $4 \cdot \text{-}3 = (\text{-}3) + (\text{-}3) + (\text{-}3) + (\text{-}3) = \text{-}12$. Also, an important property of multiplication is that we can multiply numbers in either order. This means that $\text{-}3 \cdot 4 = 4 \cdot \text{-}3 = \text{-}12$.

What about $\text{-}3 \cdot \text{-}4$? It may seem strange, but the answer is 12. To understand why this is, we can think of -4 as $(0 - 4)$.

$$(\text{-}3) \cdot (\text{-}4)$$

$$(\text{-}3) \cdot (0 - 4)$$

$$(\text{-}3 \cdot 0) - (\text{-}3 \cdot 4)$$

$$0 - \text{-}12$$

$$12$$

After more practice, your student will be able to remember this without needing to think through examples:

- A positive number times a negative number results in a negative number.
- A negative number times a positive number results in a negative number.
- A negative number times a negative number results in a positive number.

Here is a task to try with your student:

1. Calculate $5 \cdot \text{-}2$.
2. Use your answer to the previous question to calculate these products:
    a. $\text{-}2 \cdot 5$
    b. $\text{-}2 \cdot \text{-}5$
    c. $\text{-}5 \cdot \text{-}2$

Solution:

1. The answer is -10. We can think of $5 \cdot \text{-}2$ as 5 groups of -2, so
   $5 \cdot \text{-}2 = (\text{-}2) + (\text{-}2) + (\text{-}2) + (\text{-}2) + (\text{-}2) = \text{-}10$

2.

a. The answer is -10. We can multiply numbers in either order, so $-2 \cdot 5 = 5 \cdot -2 = -10$

b. The answer is 10. We can think of -5 as $(0 - 5)$, and $-2 \cdot (0 - 5) = 0 - -10 = 10$.

c. The answer is 10. Possible strategies:

- We can think of -2 as $(0 - 2)$, and $-5 \cdot (0 - 2) = 0 - -10 = 10$.
- We can multiply numbers in either order, so $-5 \cdot -2 = -2 \cdot -5 = 10$.

Section E: Four Operations with Rational Numbers

This week your student will use what they know about negative numbers to solve equations.

- The **opposite** of 5 is -5, because $5 + -5 = 0$. This is also called the additive inverse.
- The *reciprocal* of 5 is $\frac{1}{5}$, because $5 \cdot \frac{1}{5} = 1$. This is also called the multiplicative inverse.

Thinking about opposites and reciprocals can help us solve equations. For example, what value of x makes the equation $x + 11 = -4$ true?

$$\begin{aligned} x + 11 &= -4 \\ x + 11 + -11 &= -4 + -11 \\ x &= -15 \end{aligned}$$

11 and -11 are opposites.

The **solution** is -15.

What value of y makes the equation $\frac{-1}{3}y = 6$ true?

$$\begin{aligned} \frac{-1}{3}y &= 6 \\ -3 \cdot \frac{-1}{3}y &= -3 \cdot 6 \\ y &= -18 \end{aligned}$$

$\frac{-1}{3}$ and -3 are reciprocals.

The solution is -18.

Here is a task to try with your student:

Solve each equation:

$25 + a = 17$ $\quad$ $-4b = -30$ $\quad$ $\frac{-3}{4}c = 12$

Solution:

1. -8, because $17 + -25 = -8$
2. 7.5, because $\frac{-1}{4} \cdot -30 = 7.5$. (Equivalent forms of this number are also acceptable solutions.)
3. -16, because $\frac{-4}{3} \cdot 12 = -16$