Unit 7, Lesson 9

Modeling with Systems of Inequalities in Two Variables

Goals

- Define the constraints in a situation, and create a mathematical model to represent them.
- Interpret a mathematical model, presented as inequalities and graphs, that represents a situation.

Learning Targets

- I can interpret inequalities and graphs in a mathematical model.
- I know how to choose variables, specify the constraints, and write inequalities to create a mathematical model.

Lesson Narrative

In this optional culminating lesson, students integrate the ideas from the unit and engage in multiple aspects of mathematical modeling (MP4).

In the first activity, they interpret and analyze given models that represent the constraints and conditions in a situation. In the second activity, they create their own models after specifying quantities of interest, identifying relevant information, and setting the constraints.

Standards

Addressing **HSA-CED.A.3, HSA-REI.D.12, HSN-Q.A.2**

Instructional Routines

- Aspects of Mathematical Modeling
- MLR6: Three Reads
- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Graphing technology: Activity 3
- Tools for creating a visual display: Activity 3

Required Preparation

Activity 3:

Acquire devices that can run Desmos (recommended) or other graphing technology. It is ideal if students each have their own device. (Desmos is available under Math Tools.)

Student Facing Learning Goals

Let's create mathematical models using systems of inequalities.

9.1 An Enjoyment Quotient

Warm-up

10 min

Activity Narrative

This *Warm-up* gives students a chance to get an idea of how to add subjective values to a modeling situation. In later activities for this lesson, students model planning a trip using different methods of transit. One consideration for the methods of transit is which method is generally preferred. Students should keep their values in the table for an activity later in the lesson.

Standards

Addressing **HSN-Q.A.2**

Launch

Tell students that they are going to explore different methods of transit, such as riding the bus, renting an electric bike, or walking.

Ask students whether they would prefer to take a bus or walk. Would a trip in which they take the bus halfway and walk the rest be as good as one in which they ride the bus 80% of the way and walk the rest?

Display the table, and give students 1–2 minutes to notice and wonder about what they see. Then, invite students to share their observations and questions.

Tell students, "Jada has assigned values to her level of enjoyment with different modes of transit. Walking is set at a baseline level of 1, and the other methods are given values relative to the baseline. For example, Jada enjoys riding a bike twice as much as walking and being in a bus half as much as walking."

Ensure that students understand that greater numbers mean that those methods are preferred over methods with lower numbers.

Student Task Statement

Complete the table with your own relative values, giving walk the same baseline value of 1.

	Jada's enjoyment per mi	your enjoyment per mi
bus	0.5	
train	1.5	
bike rental	2	
scooter rental	0.8	
walk	1	1
car	1.3	

Student Response

Answers vary.

Activity Synthesis

Display a table with additional columns, and invite 2–4 students to share their responses. Clarify their understanding by asking questions such as:

- "Why do you think this student gave this mode of transit a value greater than 1?" (They like using that mode of transit more than walking.)
- "Did anyone use a value less than 1? What does that mean?" (Yes. It means that they like walking more than that mode of transit.)
- "Could you rank Jada's favorite methods of transit from least favorite to most?" (Yes. She likes riding the bus least, then riding a scooter a little more, then walking, then taking a car, then the train, and likes riding a bike the most.)
- "What considerations did you use to rank your enjoyment riding a bus?" (On the one hand, it is better for the environment to use public transit, and it is easy to ride without having to pay attention. On the other hand, it doesn't go directly where I want, and I'd prefer not to be around so many people.)

9.2 Custom Trip

20 min

Activity Narrative

In this activity, students use their insights from the unit to analyze and interpret a set of mathematical models and a set of data in context. Each situation involves more than two constraints, and can therefore be represented with a system with more than two inequalities.

Interpreting and connecting the inequalities, the graphs, and the data set (which involves decimals) prompts students to make sense of problems and persevere in solving them (MP1), and prompts them to reason quantitatively and abstractly (MP2).

Access for English Language Learners

This activity uses the *Three Reads* math language routine to support reading comprehension and sense making.

Standards

Addressing **HSA-REI.D.12**

Instructional Routines

- Aspects of Mathematical Modeling
- MLR6: Three Reads

Launch

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and the diagram, without revealing the questions.

- For the first read, read the problem aloud, and then ask, "What is this situation about?" (transport to a destination).

Listen for and clarify any questions about the context.

- After the second read, ask students to list any quantities that can be counted or measured (different transportation methods, cost, time, emissions, enjoyment).
- After the third read, reveal the question and Tyler's inequalities, and ask, "What are some ways we might get started on this?" Invite students to name some possible starting points, referring to quantities from the second read (compare the values listed in the inequalities to values given in the table).

To help students interpret the variables in the given inequalities as representing the number of miles traveled on each mode of transit, ask them to use the table to write an expression to represent the total emissions if they went t miles on the train and s miles on a scooter. Students should see that the expression is $125t + 202s$.

Ask for an expression representing the total enjoyment score Tyler would have for the same distances on those modes of transit ($2.2t + 1.5s$).

Arrange students in groups of 2. Ask them to analyze and answer the questions about one student's trip (either Tyler's or Jada's). If time permits, the groups could analyze the other trip.

Give students a few minutes of quiet work time and time to share their thinking with their partner. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, highlight values from the table, equations, and graphs in corresponding colors as they discover the connections. Encourage students to label regions of the graph and label variables in addition to color coding to reinforce connections.
Supports accessibility for: Visual-Spatial Processing

Student Task Statement

Here is some information about different types of travel in the city where Tyler and Jada live.

	cost	minutes per mi	emissions per mi (g CO_2 per mi)	Tyler's enjoyment per mi	Jada's enjoyment per mi
bus	\$2.50	4.6	660	1.2	0.5
train	\$2.50	3	125	2.2	1.5
bike rental	\$20	4	0	1.3	2
scooter rental	\$1 to start then \$0.80 per mi	4	202	1.5	0.8
walk	\$0	20	0	1	1
car	\$0.13	2	375	2	1.3

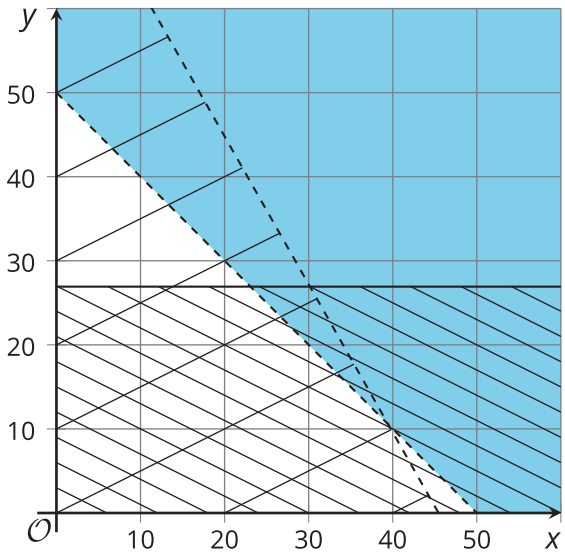
Tyler and Jada each choose their own methods of transport using two of these options. They write inequalities and create graphs to represent their constraints.

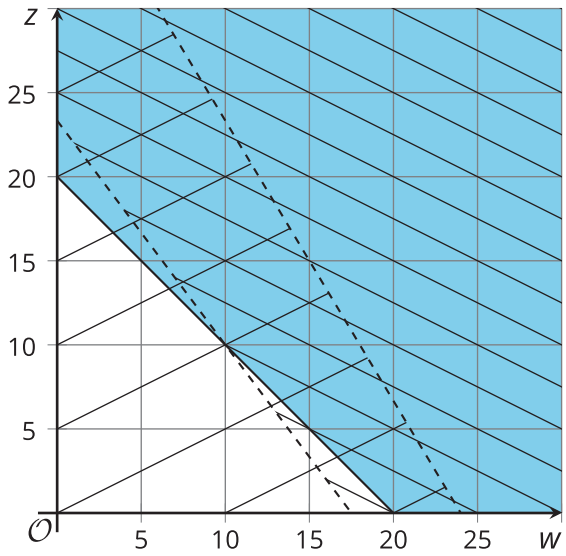
Tyler

- $x + y > 50$
- $2.50 + 0x + 0.13y \leq 6$
- $660x + 375y < 30{,}000$
- $x > 0$
- $y > 0$

Jada

- $w + z > 20$
- $2w + 1.5z > 35$
- $5w + 3z < 120$
- $w > 0$
- $z > 0$





Use the inequalities and graphs to answer these questions about each student's travel methods. Be prepared to explain your reasoning.

1. Which modes of transport did they choose?
2. What do their variables represent?
3. What does each constraint mean?
4. Which region of the graph represents which constraint?
5. Name one possible combination of values for the variables that satisfy all the constraints.

Student Response

Tyler:

1. Bus and car
2. x represents the distance traveled on the bus, and y represents the distance traveled in the car.
3. The trip is at least 50 miles, should cost less than \$6, and should emit less than 30,000 grams of carbon dioxide. The distances on each mode of transit are both positive.
4. The region with a dashed line and solid shading represents $x + y > 50$. The region with the solid, horizontal line and denser line shading represents $2.50 + 0x + 0.13y \leq 6$. The region with the dashed line and sparser line shading represents $660x + 375y < 30{,}000$.
5. Sample response: 30 miles on the bus and 23 miles in the car

Jada:

1. Bike rental and train
2. w represents the number of miles on the bike and z represents the number of miles on the train.
3. The trip is at least 20 miles, must have a enjoyment score of at least 35, and take less than 120 minutes. The distance on each mode of transit is positive.
4. The region with a solid line and solid shading represents $w + z \geq 20$. The region with the dashed line and denser line shading represents $2w + 1.5z > 35$. The region with a dashed line and sparser line shading represents $5w + 3z < 120$.
5. Sample response: 10 miles on the bike and 15 miles on the train

Activity Synthesis

Focus the discussion on the connections between the graphs and the inequalities, and on the last two inequalities for each trip. Ask questions such as:

- "How did you know which modes of transit each person used?" (By matching the coefficients in two of the inequalities to the values in the table.)
- "Why do you think Jada and Tyler both included the inequalities in which the variables are greater than zero?" (The distance on each mode of transit cannot be negative. They each used two modes of transit, so the distance on each must also not be zero.)
- "How do those inequalities affect the graph of the solution region?" (They limit the solution region to the first quadrant.)
- "Jada and Tyler each wrote five inequalities. Could all five form a single system?" (Yes) What does it mean to have a system with five inequalities?" (There are five constraints that must be met. The solutions to the system satisfy all five constraints simultaneously.)

9.3 Design Your Own Trip

20 min

Activity Narrative

This activity is designed to give students opportunities to use their understandings from this unit to perform mathematical modeling.

The travel context is familiar from the previous activity, but students are challenged to choose quantities, determine how to represent them, interpret and reason about them, and use the model they create to make choices. It also enables students to reflect on their model and revise it as needed (MP4).

Students are likely to want to use graphing technology because the information involves decimals and the inequalities written would be inconvenient to graph by hand. This is an opportunity for students to choose tools strategically (MP5).

Standards

Addressing **HSA-CED.A.3, HSA-REI.D.12, HSN-Q.A.2**

Instructional Routines

- Aspects of Mathematical Modeling
- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–4. Provide access to Desmos or other graphing technology.

Explain the expectations for researching transit values, for collaboration with group members, and for presentation of student work. (If each group is presenting one response, provide each group with tools for creating a visual display. If each student is presenting a response, give each student tools for creating a visual display.)

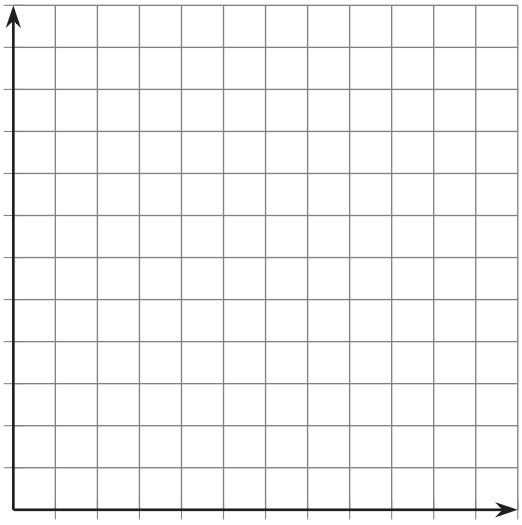
Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide reminders or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, for each question, allot a specific amount of work time and display expectations. Display a countdown timer, along with a bulleted list of what teams or individuals are to produce to complete a given step. For instance, during Question 1, display a 5-minute countdown timer and the list: "You're finished if you. . . (1) Have your two modes of transit (2) Have the transit information ready." *Supports accessibility for: Attention, Social-Emotional Functioning*

Student Task Statement

It's time to design your own trip!

1. Choose two modes of transit that you like. (You can choose from the options in the previous activity, or you can look up information for other methods.)
2. Think about the constraints for your trip. What do you want to be true about its total distance, speed, emissions, or happiness? Feel free to research additional constraints that you are interested in and that can be expressed as a per mile rate.
3. Write inequalities to represent your constraints. Then graph the inequalities.



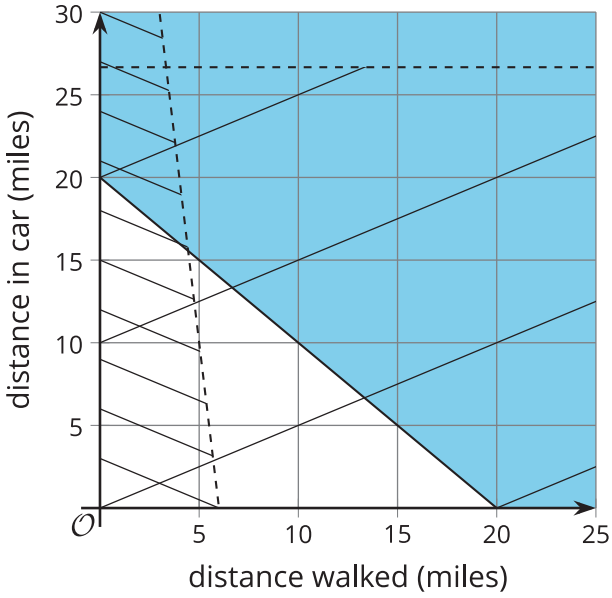
4. Is it possible to plan a trip that meets all of your constraints? If not, make changes to your constraints or your transit methods, and record them here.
5. Write a possible combination of distances on each method of transit that satisfies all the constraints.

Create a display explaining your work.

Student Response

Sample response:

1. Walk and car
2. The trip should be at least 20 miles, take less than 2 hours (120 minutes), and produce less than 10 kg (10,000 g) of carbon dioxide.
3. Let x represent the number of miles walked, and y represent the number of miles driven in the car.
   $x + y \geq 20$
   $20x + 2y < 120$
   $375y < 10,000$
   $x > 0$
   $y > 0$



4. Yes
5. Walk 2 miles and drive 21 miles.

Activity Synthesis

Select groups to share their visual displays. Encourage students to ask questions about the mathematical thinking or design approach that went into creating the display. Here are questions for discussion, if not already mentioned by students:

- "What constraints did every group use?" (All groups used that their variables must be positive.)
- "How do the graphs of the various trips compare?" (They all have an overlapping region, but some use more constraints than others.)
- "Did anyone have to revise or change their model in order to come up with a solution they could use?" (Yes, we didn't know what number to use for emissions, so we adjusted that number until it created a region that made sense.)
- "How did you use the graph to find a plan that works for the conditions of your trip?" (We looked at the overlapping shaded regions and tried to find a grid point in that region.)

Access for English Language Learners

MLR7 Compare and Connect. Invite groups to prepare a visual display that shows the strategy that they used to plan their methods of travel. Encourage students to include details that will help others interpret their thinking. Examples might include using specific language, different colors, shading, arrows, labels, notes, diagrams, or

drawings. Give students time to investigate each others' work. During the whole-class discussion, ask students, "What do the approaches have in common? How are they different?"
Advances: Representing, Conversing

Lesson Synthesis

Allow enough time for students to present their trips. Consider a gallery walk as a way for students to share their display and to ask and answer questions.

Lesson 9 Practice Problems

1 from Unit 7, Lesson 8

Student Task Statement

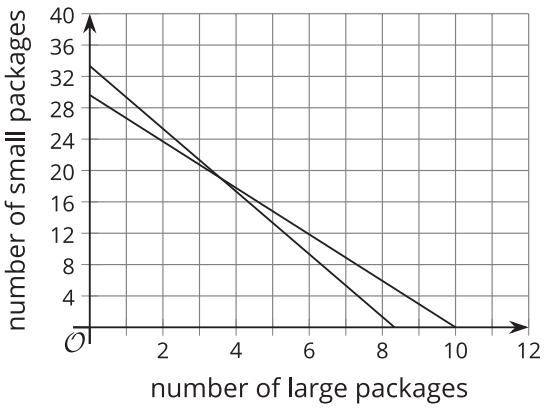
The organizer of a conference needs to prepare at least 200 notepads for the event and has a budget of $160 for the notepads. A store sells notepads in packages of 24 and packages of 6.

This system of inequalities represents these constraints: $\begin{cases} 24x + 6y \geq 200 \\ 16x + 5.40y \leq 160 \end{cases}$

a. Explain what the second inequality in the system tells us about the situation.

b. Here are incomplete graphs of the inequalities in the system, showing only the boundary lines of the solution regions.

Which graph represents the boundary line of the second inequality?



c. Complete the graphs to show the solution set to the system of inequalities.

d. Find a possible combination of large and small packages of notepads that the organizer could order.

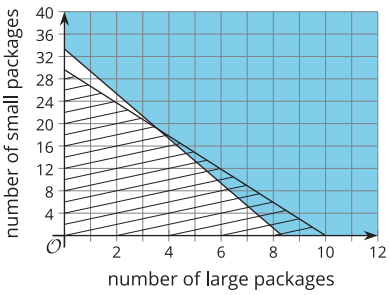
Solution

a. Sample response: The price for each large package is $16 and the price for a small package is $5.40. The total cost of buying x large packages and y small packages must be at most $160.

b. The graph that intersects the horizontal axis at 10 represents the second inequality.

c. See graph.

d. Sample response: 10 large packages and 0 small packages



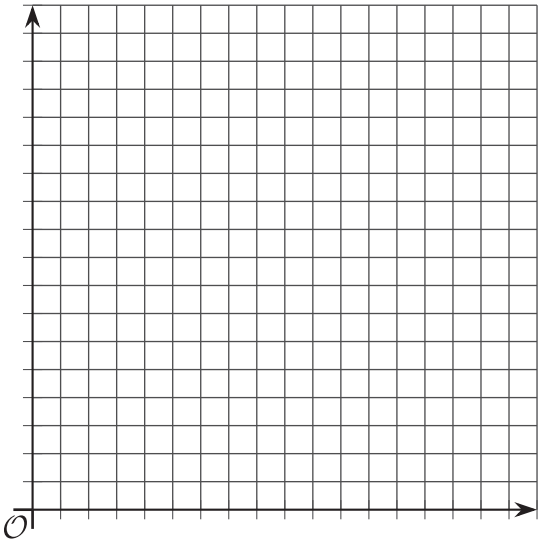
2 from Unit 7, Lesson 8

Student Task Statement

A certain stylist charges $15 for a haircut and $30 for hair coloring. A haircut takes on average 30 minutes, while coloring takes 2 hours. The stylist works up to 8 hours in a day, and she needs to make a minimum of $150 a day to pay for her expenses.

a. Create a system of inequalities that describes the constraints in this situation. Be sure to specify what each variable represents.

b. Graph the inequalities, and show the solution set.



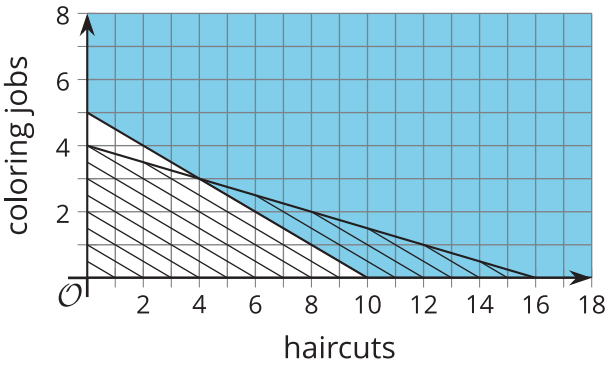
c. Identify a point that represents a combination of haircuts and hair-coloring jobs that meets the stylist's requirements.

d. Identify a point that is a solution to the system of inequalities but is not possible or not likely in the situation. Explain why this solution is impossible or unlikely.

Solution

a. Let x represent the number of haircuts and y the number of coloring appointments. $\begin{cases} 15x + 30y \geq 150 \\ 0.5x + 2y \leq 8 \end{cases}$

b. Sample graph:



c. Sample response: The point $(11, 1)$ represents 11 haircuts and 1 coloring job, which will take less than 8 hours and generate more than \$150 in income.

d. Sample response:
- The point $(12.5, 0.25)$ is in the solution region of the system of inequalities, but it is not possible to have a fractional number of haircut or hair-coloring appointments.
- The point $(13, -1)$ is a solution to the system, but it is impossible to have a negative number of hair-

coloring jobs.

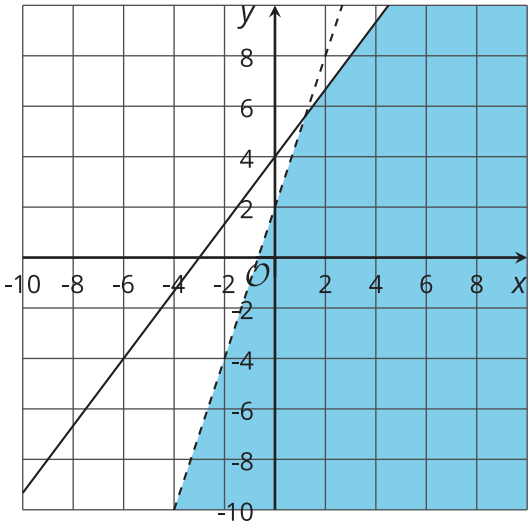
- The point $(16, 0)$ is a solution to the system, but it is very unlikely that the stylist does 16 haircuts for 8 hours without taking a break.

3 from Unit 7, Lesson 7

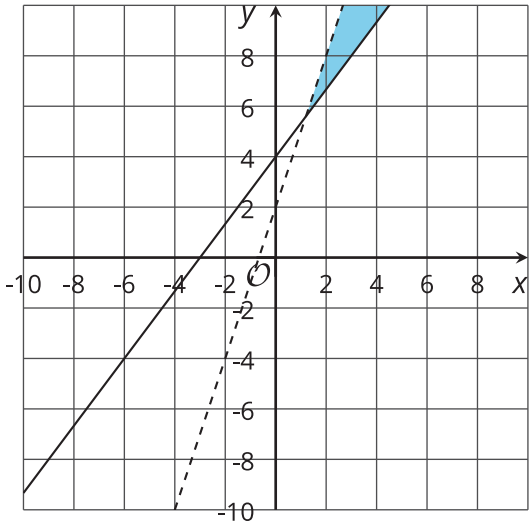
Student Task Statement

Choose the graph that shows the solution to this system: $\begin{cases} y > 3x + 2 \\ -4x + 3y \leq 12 \end{cases}$

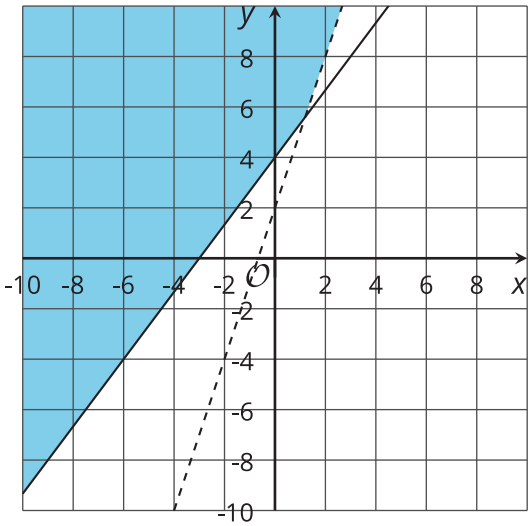
A



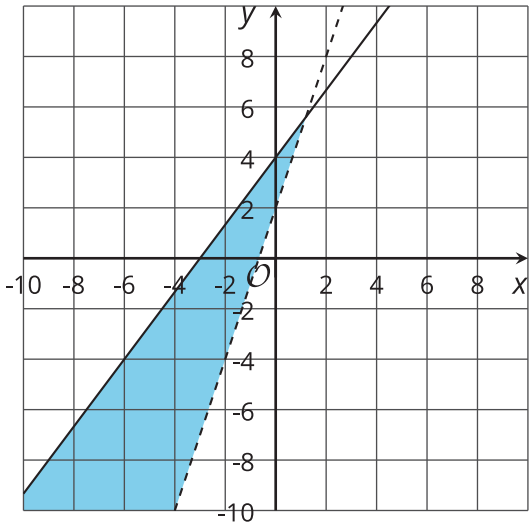
B



C



D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

Solution

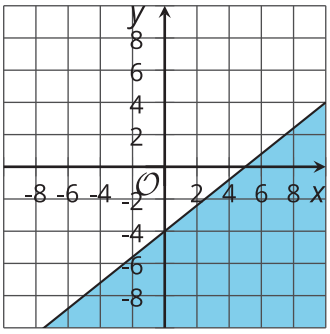
D

4 from Unit 7, Lesson 6

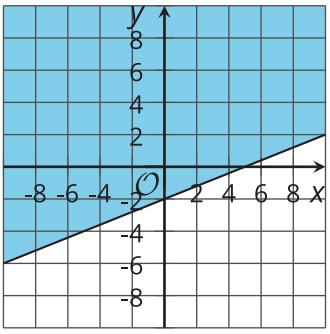
Student Task Statement

Match each inequality to the graph of its solution.

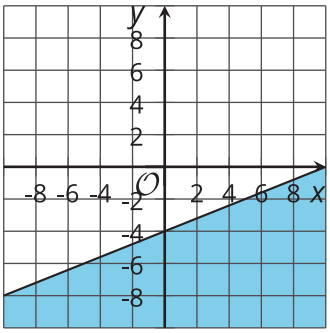
A



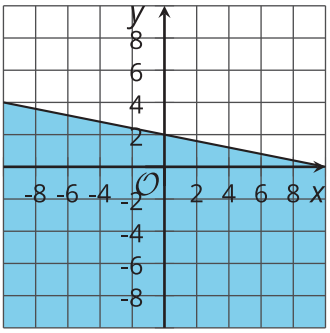
B



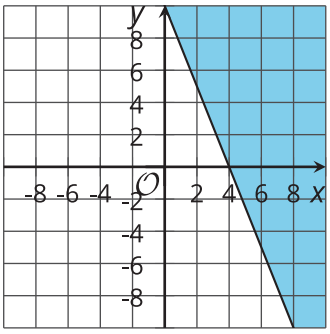
C



D



E



A. Graph A

B. Graph B

C. Graph C

D. Graph D

E. Graph E

1. $2x - 5y \geq 20$
2. $5x + 2y \geq 20$
3. $4x - 10y \leq 20$
4. $4x - 5y \geq 20$
5. $2x + 10y \leq 20$

Solution

- A matches 4
- B matches 3
- C matches 1
- D matches 5
- E matches 2