Subtraction in Equivalent Expressions

Goals

- Explain (orally, in writing, and using other representations) how the distributive and commutative properties apply to expressions with negative coefficients.
- Justify (orally and in writing) whether expressions are equivalent, including rewriting subtraction as adding the opposite.

Learning Targets

- I can organize my work when I use the distributive property.
- I can rewrite subtraction as adding the opposite and then rearrange terms in an expression.

Lesson Narrative

In this lesson, students begin working with equivalent expressions that involve negative numbers. They apply what they have learned about rewriting subtraction as adding the opposite. This enables students to apply properties of addition to generate equivalent expressions. For example, $6x - 5 + 2x$ can be rewritten as $6x + \text{-}5 + 2x$ and then rearranged as $6x + 2x + \text{-}5$ using the commutative property of addition. Then students use a similar strategy to make sense of the distributive property of multiplication over subtraction. As students rewrite an expression in different ways, they make use of structure (MP7).

Standards

Building On	**7.NS.A.1.c**
Addressing	**7.EE.A.1, 7.NS.A.1, 7.NS.A.1.c**
Building Toward	**7.EE.A.1**

Instructional Routines

- Math Talk
- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Student Facing Learning Goals

Let's find ways to work with subtraction in expressions.

1.1 Math Talk: Adding and Subtracting

Warm-up

5 min

Activity Narrative

This *Math Talk* focuses on adding and subtracting signed numbers. It encourages students to think about rewriting subtracting as adding and to rely on the properties of operations to mentally solve problems. The fluency elicited here will be helpful later in the lesson when students need to rewrite subtraction as adding the opposite.

To rewrite expressions using a different operation, students need to look for and make use of structure (MP7).

Standards

Addressing **7.NS.A.1**

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with access to sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Find the value of each expression mentally.

- $64 - 9$
- $64 + -9$
- $-9 + 64$
- $-9 - 64$

Student Response

- 55. Sample reasoning: $64 - 10 = 54$ and $54 + 1 = 55$.
- 55. Sample reasoning: Adding -9 is the same as subtracting 9.
- 55. Sample reasoning: Addition is commutative.
- -73. Sample reasoning: Subtracting 64 is the same as adding -64. This can be represented on the number line with an arrow pointing left from 0 to -9 and another arrow pointing left from -9 and going down 64 more.

Activity Synthesis

To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"

- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

Emphasize that "Subtract 10" can be rewritten as "Add negative 10" and that addition is commutative, but subtraction is not. Mention these points even if students do not bring them up.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ________ because . . . ." or "I noticed ________ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

1.2 A Helpful Observation

15 min

Activity Narrative

In this activity, students rewrite a subtraction operation as adding the opposite. Then they rearrange terms in an expression that involves only addition. This concept is applied to get students used to the idea that the subtraction sign has to stay with the term it is in front of. Making this concept explicit through a numeric example will help students see its usefulness and help them avoid common errors in working with expressions that involve subtraction.

To explain why it is valid to rewrite terms in a different order, students need to attend to precision (MP6).

Standards

Building On **7.NS.A.1.c**

Building Toward **7.EE.A.1**

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Display the expression $7\frac{3}{4}+3\frac{5}{6}-1\frac{3}{4}$ and ask students to evaluate. After they have had a chance to think about the expression, read through the *Task Statement* together before setting students to work.

Access for Students with Disabilities

Representation: Access for Perception. Invite a couple of students to act out the dialogue between Lin and Kiran. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Language, Attention

Student Task Statement

Lin and Kiran are trying to calculate $7\frac{3}{4}+3\frac{5}{6}-1\frac{3}{4}$. Here is their conversation:

Lin: "I plan to first add $7\frac{3}{4}$ and $3\frac{5}{6}$, so I will have to start by finding equivalent fractions with a common denominator."

Kiran: "It would be a lot easier if we could start by working with the $1\frac{3}{4}$ and $7\frac{3}{4}$. Can we rewrite it like $7\frac{3}{4} + 1\frac{3}{4} - 3\frac{5}{6}$?"

Lin: "You can't switch the order of numbers in a subtraction problem like you can with addition. $2 - 3$ is not equal to $3 - 2$."

Kiran: "That's true, but do you remember what we learned about rewriting subtraction expressions using addition? $2 - 3$ is equal to $2 + (-3)$."

1. Write an expression with three **terms** that is equivalent to $7\frac{3}{4} + 3\frac{5}{6} - 1\frac{3}{4}$ and uses only addition.
2. If you wrote the terms of your new expression in a different order, would it still be equivalent? Explain your reasoning.

Student Response

1. $7\frac{3}{4} + 3\frac{5}{6} + \left(-1\frac{3}{4}\right)$ (or equivalent)
2. Sample response: It works as long as the subtraction or negative sign is moved along with the number that follows. What doesn't work is moving the numbers but leaving the subtraction sign in the same place.

Activity Synthesis

The purpose of this discussion is to review rewriting subtraction as "adding the opposite," and to reinforce understanding that rewriting an expression using addition means that it's valid to rearrange the terms.

Ensure everyone agrees that $7\frac{3}{4} + 3\frac{5}{6} - 1\frac{3}{4}$ is equivalent to $7\frac{3}{4} + 3\frac{5}{6} + \left(-1\frac{3}{4}\right)$ is equivalent to $7\frac{3}{4} + -1\frac{3}{4} + 3\frac{5}{6}$. Use the language "commutative property of addition."

Access for English Language Learners

MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to "If you wrote the terms of your new expression in a different order, would it still be equivalent? Explain your reasoning." Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

15 min

1.3 Organizing Work

Activity Narrative

In this activity, students express the area of a rectangle in different ways using the distributive property. Students learn that even when the terms are negative, we can still organize our work with the distributive property in a way that is similar to using area diagrams.

Students must notice and make use of structure to rewrite an expression in different ways (MP7).

Standards

Building On **7.NS.A.1.c**

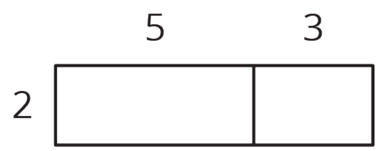
Addressing **7.EE.A.1**

Instructional Routines

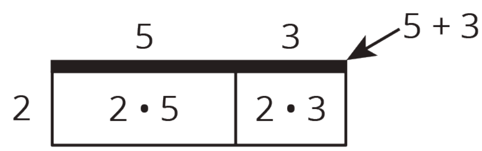
- MLR8: Discussion Supports

Launch

Display the image, and ask students to write an expression for the area of the big rectangle in at least three different ways.



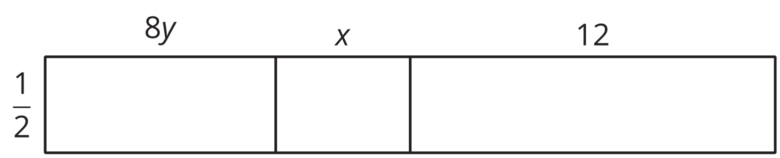
Collect responses. If students simply say "16," ask them to explain how they calculated 16 and record these processes for all to see. Remind students that thinking about area gives us a way to understand the distributive property. This diagram can be used to show that $2 \cdot 5 + 2 \cdot 3 = 2(5+3)$. Be sure that students see you write the partial products in the diagram, and that they see every piece of the associated identity $2 \cdot 5 + 2 \cdot 3 = 2(5+3)$



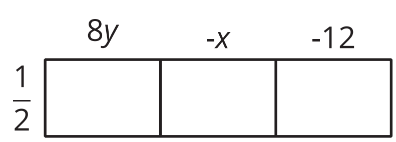
Tell students that when we are working with negative numbers, thinking about area doesn't work so well, but the distributive property still holds when there are negative numbers. The expressions involved still have the same structure, and we can still organize our work the same way.

Student Task Statement

1. Write two expressions for the area of the big rectangle.



2. Use the distributive property to write an expression that is equivalent to $\frac{1}{2}(8y + \text{-}x + \text{-}12)$. The boxes can help you organize your work.



3. Use the distributive property to write an expression that is equivalent to $\frac{1}{2}(8y - x - 12)$.

Student Response

Accept all equivalent forms for each answer.

1. $\frac{1}{2}(8y + x + 12)$ and $4y + \frac{1}{2}x + 6$
2. $4y + \text{-}\frac{1}{2}x + \text{-}6$
3. $4y - \frac{1}{2}x - 6$

Are You Ready for More?

Here is a calendar for April 2028.

April 2028

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

Let's choose a date: the 10th.

Look at the numbers above, below, and to either side of the 10th: 3, 17, 9, 11.

1. Average these four numbers. What do you notice?
2. Choose a different date that is in a location where it has a date above, below, and to either side. Average these four numbers. What do you notice?
3. Explain why the same thing will happen for any date in a location where it has a date above, below, and to either side.

Extension Student Response

1. The average of the four numbers is 10. Sample reasoning: The average of the four surrounding numbers equals the original date chosen: $(3+17+9+11) \div 4 = 40 \div 4 = 10$.
2. Sample response: Using April 21, the four surrounding numbers are 14, 28, 20, and 22. The average of these is 21, because $(14+28+20+22) \div 4 = 84 \div 4 = 21$. The average of the four surrounding numbers equals the original date chosen.
3. Sample response: If the original date chosen is represented by x, then the date above is $x-7$ because it must be 7 days prior. The date below is $x+7$ because it must be 7 days after. The date to the left is $x-1$ and the date to the right is $x+1$. The sum of these four dates is $x-7+x+7+x-1+x+1$ which equals $4x$. To find the average, I would divide this by 4, giving the original date chosen, x.

Activity Synthesis

The purpose of this discussion is to recognize and justify different, valid ways to write equivalent expressions.

Solicit responses to the second question and demonstrate thinking about one product at a time:

	$8y$	$-x$	-12
$\frac{1}{2}$	$4y$	$-\frac{1}{2}x$	-6

Then ask students to share how they approached the last question. Highlight responses where students noticed that $\frac{1}{2}(8y-x-12)$ can be rewritten like $\frac{1}{2}(8y+-x+-12)$ because of what they talked about in the *Warm-up*. So, the two questions could have the same answer.

Access for English Language Learners

MLR8 Discussion Supports. For each observation that is shared, invite students to turn to a partner and restate what they heard, using precise mathematical language.
Advances: Listening, Speaking

Lesson Synthesis

Share with students, "Today we saw how rewriting subtraction as adding the opposite can help us write equivalent expressions."

If desired, use this example to review these concepts. Display the expressions $x + 2 - 3x - 10$ and $x + 3x - 2 - 10$ for all to see. Ask:

- "Are these expressions equivalent? How do you know?" (No. Subtraction is not commutative. $2 - 3x$ is not the same as $3x - 2$.)
- "How could we fix the second expression to make it equivalent to the first, without moving the terms?" (Put the subtraction sign in front of the $3x$ instead of the 2.)

1.4 Equivalent to $4 - x$

Cool-down

5 min

Standards

Addressing **7.EE.A.1, 7.NS.A.1.c**

Student Task Statement

1. Select **all** the expressions that are equivalent to $4 - x$.

   A. $x - 4$

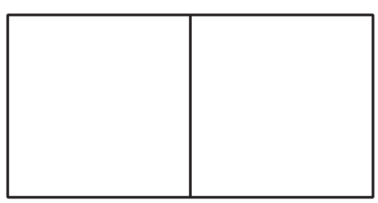
   B. $4 + -x$

   C. $-x + 4$

   D. $-4 + x$

   E. $4 + x$

2. Use the distributive property to write an expression that is equivalent to $5(-2x - 3)$. If you get stuck, use the boxes to help organize your work.



Student Response

1. B, C
2. $-10x - 15$ (or equivalent)

Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 1 Summary

In previous lessons, we learned that subtracting a number gives the same result as adding its opposite. We can apply this relationship to rewrite an expression with subtraction so it uses only addition. Then we can make use of the properties of addition that allow us to add and group in any order. This can make calculations simpler. Example:

$$\frac{5}{8} - \frac{2}{3} - \frac{1}{8}$$

$$\frac{5}{8} + (-\frac{2}{3}) + (-\frac{1}{8})$$

$$\frac{5}{8} + (-\frac{1}{8}) + (-\frac{2}{3})$$

$$\frac{4}{8} + (-\frac{2}{3})$$

We can also organize the work of multiplying signed numbers in expressions. The product $\frac{3}{2}(6y - 2x - 8)$ can be found by drawing a rectangle with the first factor, $\frac{3}{2}$, on one side, and the three **terms** inside the parentheses on the other side:

	$6y$	$-2x$	-8
$\frac{3}{2}$			

Multiply $\frac{3}{2}$ by each term across the top:

	$6y$	$-2x$	-8
$\frac{3}{2}$	$\frac{3}{2} \cdot 6y$	$\frac{3}{2} \cdot -2x$	$\frac{3}{2} \cdot -8$

	$6y$	$-2x$	-8
$\frac{3}{2}$	$9y$	$-3x$	-12

Reassemble the parts to get the expanded version of the original expression:

$$\frac{3}{2}(6y - 2x - 8) = 9y - 3x - 12$$

Glossary

- term

Lesson 1 Practice Problems

1 Student Task Statement

For each expression, write an equivalent expression that uses only addition.

a. $20 - 9 + 8 - 7$

b. $4x - 7y - 5z + 6$

c. $-3x - 8y - 4 - \frac{8}{7}z$

Solution

Sample responses:

a. $20 + -9 + 8 + -7$

b. $4x + -7y + -5z + 6$

c. $-3x + -8y + -4 + -\frac{8}{7}z$

2 Student Task Statement

Use the distributive property to write an expression that is equivalent to each expression. If you get stuck, consider drawing boxes to help organize your work.

a. $9(4x - 3y - \frac{2}{3})$

b. $-2(-6x + 3y - 1)$

c. $\frac{1}{5}(20y - 4x - 13)$

d. $8(-x - \frac{1}{2})$

e. $-8(-x - \frac{3}{4}y + \frac{7}{2})$

Solution

Sample responses:

a. $36x - 27y - 6$

b. $12x - 6y + 2$

c. $4y - \frac{4}{5}x - \frac{13}{5}$

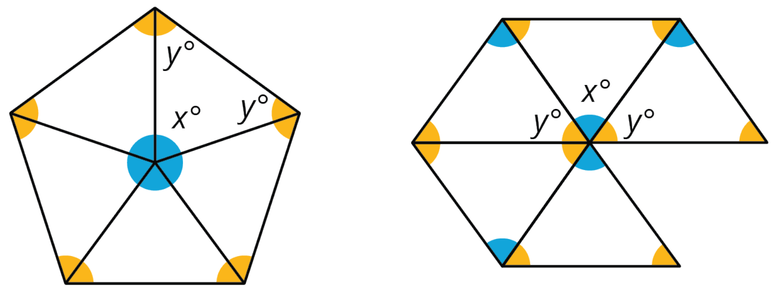
d. $-8x - 4$

e. $8x + 6y - 28$

3 from Unit 3, Lesson 18

Student Task Statement

Here are two different patterns made out of the same five identical isosceles triangles. Without using a protractor, determine the measures of Angles x and y. Explain or show your reasoning.



Solution

$x = 72$ and $y = 54$. Since there are 5 copies of the angle that measures x around a single point in the first picture, we know that $5x = 360$, so $x = 72$. In the second picture, we know that two copies of y and one copy of x make a straight angle, so $2y + 72 = 180$. Since we already know x, we can figure out that $y = 54$.

4 from Unit 3, Lesson 8

Student Task Statement

Match each equation to a step that will help solve the equation for x.

A. $3x = -4$

B. $-4.5 = x - 3$

C. $3 = \frac{-x}{3}$

D. $\frac{1}{3} = -3x$

E. $x - \frac{1}{3} = 0.4$

F. $3 + x = 8$

G. $\frac{x}{3} = 15$

H. $7 = \frac{1}{3} + x$

1. Add $\frac{1}{3}$ to each side.
2. Add $\frac{-1}{3}$ to each side.
3. Add 3 to each side.
4. Add -3 to each side.
5. Multiply each side by 3.
6. Multiply each side by -3.
7. Multiply each side by $\frac{1}{3}$.
8. Multiply each side by $\frac{-1}{3}$.

Solution

- A matches 7
- B matches 3

- C matches 6
- D matches 8
- E matches 1
- F matches 4
- G matches 5
- H matches 2

5 from an earlier course

Student Task Statement

Here are five sums. Use the distributive property to write each sum as a product with two factors.

a. $2a+7a$

b. $5z-10$

c. $c-2cd$

d. $r+r+r+r$

e. $2x-\frac{1}{2}$

Solution

Sample responses:

a. $(2+7)a$ or $9a$

b. $5(z-2)$

c. $c(1-2d)$

d. $(1+1+1+1)r$ or $4r$

e. $2(x-\frac{1}{4})$ or $\frac{1}{2}(4x-1)$