Unit 4, Lesson 1

Growing and Shrinking

Let's calculate exponential change.

1.1 Bank Accounts

A bank account has a balance of \$120 on January 1. Describe a situation in which the account balance for each month (February 1, March 1, . . .) forms these types of sequences. Write the first three terms of each sequence.

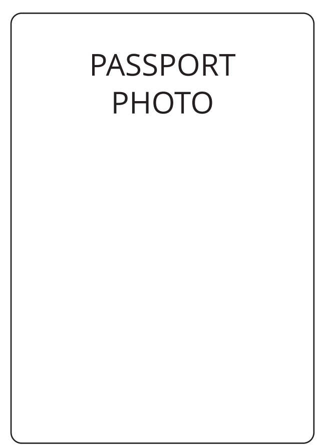
1. an arithmetic sequence
2. a geometric sequence

1.2 Shrinking a Passport Photo

Elena needs a passport photo that meets certain requirements to be used. One of the requirements is that the measurement from her chin to the top of her head is between 25 millimeters and 35 millimeters in the photo.

Elena has a photo of herself in which that measurement is 150 millimeters, and she has access to a photocopier that can reduce the height of the photo to 80% of its previous value each time it is copied.

1. Find the measurement from her chin to the top of her head after the image has been scaled by 80% these number of times. Explain or show your reasoning.
   a. 3 times
   b. 6 times
2. How many times would the image need to be scaled by 80% for that measurement to be less than 35 mm?
3. How many times would the image need to be scaled by 80% for it to be less than 25 mm?



Are you ready for more?

Suppose you'd like to rescale the passport image that has been scaled down 7 times back to its original size by scaling only once. At what percentage should you set the scale on the image editor?

1.3 Pond in a Park

On May 12, a fast-growing species of algae is accidentally introduced into a pond in a park. The area of the pond that the algae covers doubles each day. If not controlled, the algae will cover the entire surface of the pond, depriving the fish in the pond of oxygen. At the rate it is growing, this will happen on May 24.

1. On which day is the pond halfway covered?
2. On May 18, Clare visits the park. A park caretaker mentions to her that the pond will be completely covered in less than a week. Clare looks at how much of the pond is currently covered and thinks that the caretaker must be mistaken. Why might she find the caretaker's claim hard to believe?
3. What fraction of the area of the pond was covered by the algae initially, on May 12? Explain or show your reasoning.

Lesson 1 Summary

Sometimes quantities change by the same factor at regular intervals.

For example, a bacteria population might be 10,000 on the day of measurement and then double each day after that point. This means that one day after the population is measured, the population would be 20,000, two days after the measurement, it would be 40,000, and three days after, it would be 80,000.

The relationship can be modeled by an exponential function because the population changes by the same factor for each passing day. If n is the number of days since the bacteria population was first measured, then the population on day n is $10{,}000 \cdot 2^n$. The population each day is also a geometric sequence because each term is found by multiplying the previous term by 2.

days since population is measured	population
0	10,000
1	$10{,}000 \cdot 2$
2	$10{,}000 \cdot 2^2$
3	$10{,}000 \cdot 2^3$
n	$10{,}000 \cdot 2^n$