Solutions to Linear Equations

Goals

- Create a graph and an equation in the form $Ax + By = C$ that represent a linear relationship.
- Describe that for the graph and equation of the same line, every point on the line is a solution to the equation, and any point not on the line is not a solution to the equation.
- Determine pairs of values that satisfy or do not satisfy a linear relationship using an equation or graph.

Learning Targets

- I know that the graph of an equation is a visual representation of all the solutions to the equation.
- I understand what the solution to an equation in two variables is.

Lesson Narrative

In this lesson, students find **solutions to an equation with two variables**: pairs of numbers that make the equation and situation true. They look at contexts where both variables have to satisfy a constraint, often represented with an equation of the form $Ax + By = C$.

Students consider two situations. The first looks at different ways of spending $10 on two differently priced fruits. The second looks at pairs of numbers where twice the first number plus the second number adds up to 10. While the constraints for these two situations result in two equivalent equations, creating the graphs representing each requires students to interpret the meaning of their solutions in a context (MP2). For example, the graph representing the two numbers is a straight line that includes positive and negative numbers, while the graph representing the fruit is a set of discrete points that lie on the same line and must be positive whole numbers.

Students also consider pairs of numbers that do not make the equation or situation true, observing that those points do not lie on the graph.

Standards

Building On	**6.G.A.1**
Addressing	**8.EE.B, 8.EE.C**
Building Toward	**8.EE.C**

Instructional Routines

- 5 Practices
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Colored pencils: Activity 3
- Graph paper: Activity 3
- Straightedges: Activity 3

Student Facing Learning Goals

Let's think about what it means to be a solution to a linear equation with two variables in it.

13.1 Avocados and Pineapples

Warm-up

5 min

Activity Narrative

The purpose of this activity is to activate student thinking around expressions and equations with two variables by considering the cost of different combinations of 2 fruits. This will be useful later when students interpret the meaning of variables in context and reason about systems of linear equations.

Standards

Building On **6.G.A.1**

Launch

Begin by asking students to share their favorite fruits. If possible, record student responses for all to see. If necessary, explain that sometimes fruit is priced by weight and sometimes it is priced based on the number of items.

Give students 2 minutes of quiet work time followed by a partner then whole-class discussion.

Student Task Statement

At the market, avocados cost \$1 each and pineapples cost \$2 each. Find the cost of:

1. 6 avocados and 3 pineapples
2. 4 avocados and 4 pineapples
3. 5 avocados and 4 pineapples
4. 8 avocados and 2 pineapples

Student Response

1. \$12
2. \$12
3. \$13
4. \$12

Activity Synthesis

The goal of this discussion is to make sure students have a strategy for calculating the cost of different combinations of fruit. Begin by inviting students to share their strategies and record them for all to see. Consider asking:

- "What do all of these methods have in common?"

- “What do you notice about the combinations of fruit that all cost the same total amount?”
- “What do you notice about the one combination of fruit that cost a different amount than the others?”

13.2 More Avocados and Pineapples

10 min

Activity Narrative

In this activity, students continue to examine equations of the form $Ax + By = C$ as they consider combinations of fruit that keep a total cost constant. The solutions are limited to non-negative integers and the set of all solutions is finite.

Monitor for students who write the equation $a + 2p = 10$ to describe the $10 combinations of fruit. There is no reason to solve for one variable in terms of the other, since a graph has not been requested and neither variable is dependent on the other.

Standards

Addressing **8.EE.B**

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give students 3–4 minutes of quiet think time followed by partner then whole-class discussion.

Access for English Language Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations. Monitor and clarify any questions about the context.
Advances: Reading, Representing

Student Task Statement

At the market, avocados cost $1 each and pineapples cost $2 each.

1. Noah has $10 to spend at the produce market. Can he buy 7 avocados and 2 pineapples? Explain or show your reasoning.
2. What combinations of avocados and pineapples can Noah buy if he spends all of his $10?
3. Write an equation that represents $10 combinations of avocados and pineapples, using a for the number of avocados and p for the number of pineapples.
4. What are 3 combinations of avocados and pineapples that make your equation true? What are three combinations of avocados and pineapples that make it false?

Student Response

1. No. Sample reasoning: 7 avocados and 2 pineapples would cost $11.
2. 0 avocados and 5 pineapples, 2 avocados and 4 pineapples, 4 avocados and 3 pineapples, 6 avocados and 2

pineapples, 8 avocados and 1 pineapple, 10 avocados and 0 pineapples.

3. $a + 2p = 10$ (or equivalent)
4. Sample responses:
   - Combinations that make the equation true: 2 avocados and 4 pineapples, 4 avocados and 3 pineapples, 6 avocados and 2 pineapples (or any combination that adds up to $10)
   - Combinations that make the equation false: 3 avocados and 10 pineapples, 0 avocados and 7 pineapples, 9 avocados and 3 pineapples (or any combination that does not add up to $10)

Are You Ready for More?

1. Create a graph relating the number of avocados and the number of pineapples that can be purchased for exactly $10.
2. What is the slope of the graph? What is the meaning of the slope in terms of the context?
3. Suppose Noah has $20 to spend. Graph the equation describing this situation. What do you notice about the relationship between this graph and the earlier one?

Extension Student Response

1. The graph of the equation should show horizontal and vertical intercepts, first quadrant only, with the horizontal axis labeled “number of avocados” and the vertical axis labeled “number of pineapples.” Note that a graph with axes labels reversed is also correct.
2. The slope is $\frac{1}{2}$ (or -2 if the axes are swapped). This is the number of pineapples per avocado (or avocados per pineapple if the axes are swapped).
3. The graph is the same as in the first question except that the intercepts are at 20 and 10 instead of 10 and 5. The new line has the same slope, because the tradeoff between avocados and pineapples stays the same.

Activity Synthesis

The goal of this discussion is for students to see that the combinations of fruit that can be purchased for a total cost of $10 are solutions to this situation and also to the equation $a + 2p = 10$ (or some equivalent).

Invite students to share the combinations of avocados and pineapples that cost a total of $10. (There are 6 combinations.) Record their responses for all to see. Next, invite previously selected students to share their equation representing the situation and record them for all to see.

Explain to students that the 6 combinations of fruit that cost $10 are solutions to this situation and also to the equation $a + 2p = 10$. Emphasize that each solution has two values — one for avocados and one for pineapples.

Ask students:

- “Were there any patterns that helped you find the combinations?” (Buying 1 less pineapple means you can buy 2 more avocados.)
- “According to the equation you wrote, buying $\frac{1}{2}$ a pineapple and 9 avocados would also cost $10. Do you think this situation is realistic?” (Probably not, you would only buy whole pieces of fruit.)

20 min

13.3 Solutions and Everything Else

Activity Narrative

In this activity, students write an equation representing a relationship between two quantities. Note that the relationship stated here matches the relationship describing avocados and pineapples that can be purchased for $10, but without the constraints of requiring whole-number values.

Students find pairs of numbers that make the equation and stated relationship true and not true. By graphing both sets of points, they use repeated reasoning to observe that the graph of a linear equation is the set of its solutions, and any point not on this line is not a solution (MP8).

As students plot points that make the equation and stated statement true, monitor for students who create graphs with these features, sequenced in order from more common to less common:

- Points in the first quadrant only
- Points on the axes (there are only two of these!)
- Points in the second or fourth quadrants
- Points with non-integer coordinate values

Standards

Building Toward **8.EE.C**

Instructional Routines

- 5 Practices

Launch

Arrange students in groups of 2. Provide access to graph paper, a straightedge, and a different color of pen or pencil.

Display the task statement for all to see: "There are two numbers. When the first number is doubled and added to the second number, the sum is 10."

Ask the class to predict, before calculating anything, how many different pairs of numbers make the statement true. Record the responses for all to see.

Give students 8–10 minutes of quiet work time followed by a partner then whole-class discussion.

Select students with graphs that include the features described in the *Activity Narrative* to share later.

Student Task Statement

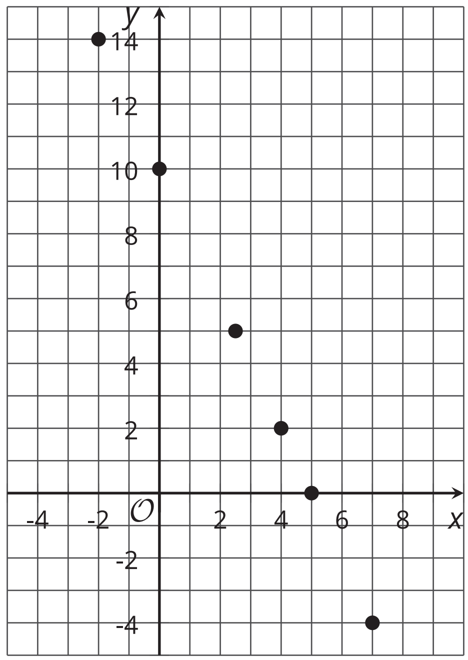
There are two numbers. When the first number is doubled and added to the second number, the sum is 10.

1. Let x represent the first number and let y represent the second number. Write an equation showing the relationship between x, y, and 10.
2. Draw and label a coordinate plane.
3. Find 5 pairs of x- and y-values that make the statement and your equation true. Plot each pair of values as a point (x, y) on the coordinate plane. What do you notice?
4. List 10 pairs of x- and y-values that do not make the statement and equation true. Using a different color, plot each pair of values as a point (x, y) on the coordinate plane. What do you notice about these points

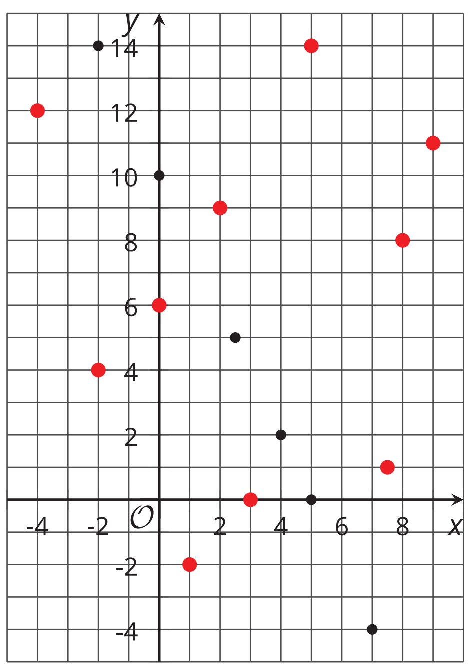
compared to your first set of points?

Student Response

1. $2x + y = 10$ (or equivalent)
2. No answer needed
3. See graph. Sample response: All the points lie on a line.



4. See graph. Sample response: None of these points lie on the same line as the set of points that made the statement true.



Building on Student Thinking

If students write $x + 2y = 10$, consider asking:

- "What do x and y represent in your equation?"
- "Which number, the first or second, gets doubled?"

Activity Synthesis

The purpose of this discussion is for students to see that values that make the relationship and equation true all lie in a line, and numbers not on the line will not make the equation or relationship true.

Invite previously selected students to share their graphs of points that make the equation and statement true. Sequence the discussion of the graphs in the order listed in the *Activity Narrative*. Create a classroom display with a graph that includes all of the points shared by students, adding additional points as each student shares. As each graph is shared, ask students:

- "What do you notice about this graph?" (The points all lie on the same line.)
- "What is something on this graph that hasn't been seen yet?" (a point that lies one of the axes; negative x- or y-values; values for x or y that are not integers)

If not brought up in students' graphs, ask if the equation could be true if x was a negative value, such as $x = -3$? If y was a negative value, such as $y = -4$? If x was not an integer such as $x = 3\frac{1}{2}$?" (Yes, the corresponding x- or y-values would be $y = 16$, $x = 7$, and $y = 3$ respectively.) If necessary, add these points to the classroom display.

After all previously selected graphs have been shared, connect the different responses to the learning goals by asking questions such as:

- "Based on your observations, what is the relationship between the solutions of an equation and its graph?" (The graph of the equation is the set of all solution pairs (x, y) plotted as points in the coordinate plane.)
- "Is there any number for x or y where this equation would not be true?" (No. For every value of x or y there will be a corresponding y- or x-value that will make the equation true.)
- "What does the graph tell about the number of solutions of your equation?" (There are an infinite number of solutions.)
- "What did you notice about the points with x- and y-values that did not make the statement true?" (Those points did not lie on the same line as the points with values that did make the statement true.)
- "What does it mean if a point (x, y) does not lie on the line for the equation $2x + y = 10$?" (It means that pair of values for x and y is not a solution to the equation.)

Access for Students with Disabilities

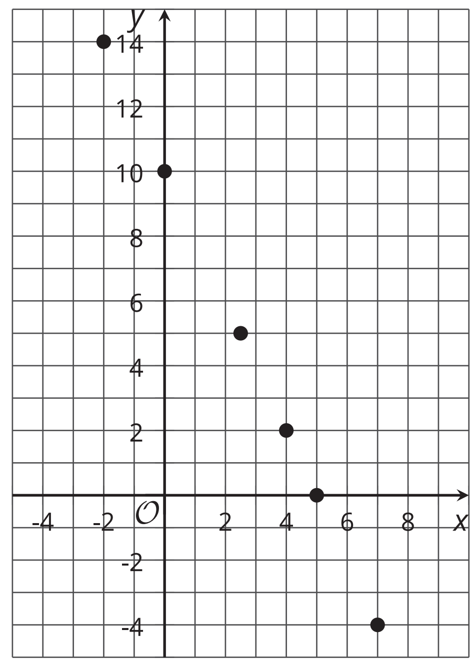
Representation: Internalize Comprehension. Use multiple examples and non-examples to emphasize relationships between pairs of numbers that satisfy the given statement. For example, the pair $(1, 8)$ makes the statement true, whereas $(2, 4)$ does not.
Supports accessibility for: Conceptual Processing, Attention

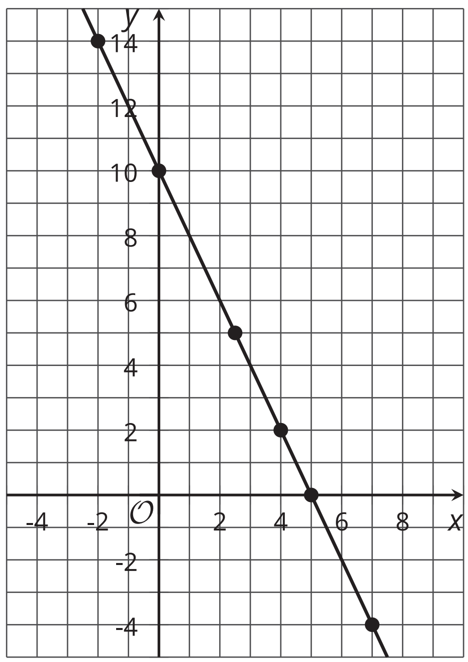
Lesson Synthesis

The goal of this discussion is for students to understand that the **solution to an equation with two variables** is a pair of values for the variables that make the equation true. Begin by displaying the equation $a + 2p = 10$ for all to see and ask students to recall what this equation represents (the number of avocados and pineapples that can be bought for $10). Explain that each possible combination of fruit is a *solution to this equation with two variables*. Emphasize that the solution consists of two values, in this case the number of avocados a and the number of pineapples p.

Then ask students what a graph representing the situation with avocados and pineapples might look like. If any students created a graph for the *Are You Ready for More* question, invite them to share their graph now. If time allows, encourage students to create a quick sketch.

Then display the equation $2x + y = 10$ and this graph, or the graph with students' points created in a previous activity, for all to see. Remind students that this graph represents all the pairs of numbers that have a sum of 10 when one is doubled.





Draw in the line that passes through all of the points and explain that the line represents all of the solutions to the equation $2x + y = 10$.

Then discuss with students:

- "How would a graph representing the situation with avocados and pineapples be similar or different from this graph representing the situation with two numbers?" (Both graphs represent solutions to the same equation. The graph representing the fruit would only contain the six points that represent the six combinations of fruit that cost \$10, while the graph representing the two numbers would be a line.)
- "Can points that are not on the line be solutions to the equation represented by the line?" (No.)
- "Can a single value be a solution to an equation with two variables?" (No, a solution is a pair of values—one for each of the variables.)

13.4 Identify the Points

Cool-down

5 min

Standards

Addressing **8.EE.C**

Student Task Statement

Select all the coordinates that represent a point on the graph of the line $x - 9y = 12$.

A. $(12, 0)$

B. $(0, 12)$

C. $(3, -1)$

D. $\left(0, -\frac{4}{3}\right)$

E. $(-3, 1)$

Student Response

A, C, D

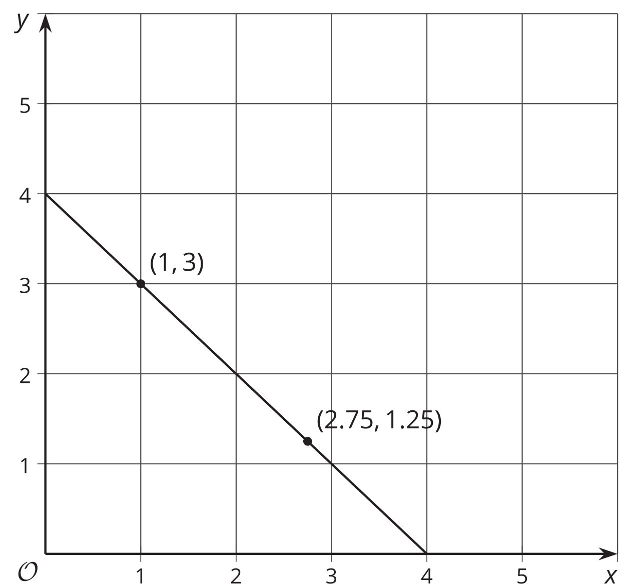
Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 13 Summary

A **solution to an equation with two variables** is any pair of values for the variables that make the equation true. For example, the equation $2x + 2y = 8$ represents the relationship between the width x and length y for rectangles with a perimeter of 8 units. One solution to the equation $2x + 2y = 8$ is that the width and length could be 1 and 3, since $2 \cdot 1 + 2 \cdot 3 = 8$. Another solution is that the width and length could be 2.75 and 1.25, since $2 \cdot (2.75) + 2 \cdot (1.25) = 8$. There are many other possible pairs of width and length that make the equation true.

The pairs of numbers that are solutions to an equation can be seen as points on the coordinate plane where every point represents a different rectangle whose perimeter is 8 units. Here is part of the line created by all the points (x, y) that are solutions to $2x + 2y = 8$. In this situation, it makes sense for the graph to only include positive values for x and y since there is no such thing as a rectangle with a negative side length.



Glossary

- solution to an equation with two variables

Lesson 13 Practice Problems

1 Student Task Statement

Select **all** of the ordered pairs (x, y) that are solutions to the linear equation $2x + 3y = 6$.

A. $(0, 2)$

B. $(0, 6)$

C. $(2, 3)$

D. $(3, -2)$

E. $(3, 0)$

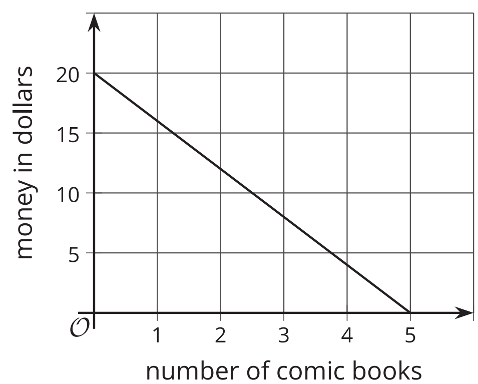
F. $(6, -2)$

Solution

A, E, F

2 Student Task Statement

The graph shows the relationship between the number of comic books Priya buys at the store and the amount of money in dollars that Priya has left after buying the comic books.



a. What is the vertical intercept and what does it mean in this situation?

b. What is the horizontal intercept and what does it mean in this situation?

c. What is the slope of this line and what does it mean in this situation?

d. Write an equation that represents this line where x represents the number of comic books Priya buys and y represents how many dollars Priya has left.

e. If Priya buys 3 comic books, how much money will she have remaining?

Solution

a. The vertical intercept is 20 and means that Priya had 20 dollars before buying any comics.

b. The horizontal intercept is 5 and means that Priya can buy 5 comic books before she runs out of money.

c. The slope is -4 and means that each comic book costs 4 dollars since the amount of money Priya has left decreases by 4 with each comic book she buys.

d. $y = 20 - 4x$ (or equivalent)

e. 8 dollars

3 Student Task Statement

Match each equation with the set of points that are all solutions to the equation.

A. $y = 1.5x$

B. $2x + 3y = 7$

C. $x - y = 4$

D. $3x = \frac{y}{2}$

E. $y = -x + 1$

1. $(14, 21), (2, 3), (8, 12)$
2. $(-3, -7), (0, -4), (-1, -5)$
3. $\left(\frac{1}{8}, \frac{7}{8}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{4}, \frac{3}{4}\right)$
4. $\left(1, 1\frac{2}{3}\right), (-1, 3), \left(0, 2\frac{1}{3}\right)$
5. $(0.5, 3), (1, 6), (1.2, 7.2)$

Solution

- A matches 1
- B matches 4
- C matches 2
- D matches 5
- E matches 3

4

from Unit 3, Lesson 10

Student Task Statement

A container of fuel empties at a rate of 5 gallons per second. A graph representing this situation is drawn in the coordinate plane, where x is the number of seconds that have passed since the container started emptying, and y is the amount of fuel remaining in the container.

Will the slope of the line representing this situation have a positive, negative, or zero slope? Explain your reasoning.

Solution

Negative. Sample reasoning: The amount of fuel remaining in the tank, y, decreases as the number of seconds dispensing fuel, x, increases.

5 from Unit 3, Lesson 5

Student Task Statement

A sandwich shop charges a delivery fee to bring lunch to an office building. One office pays 33 dollars for 4 turkey sandwiches including the delivery fee. Another office pays 61 dollars for 8 turkey sandwiches including the delivery fee.

a. What is the cost of one turkey sandwich? Explain your reasoning.

b. What is the delivery fee? Explain your reasoning.

Solution

a. One turkey sandwich costs 7 dollars. Sample reasoning: The second office pays $61 - 33 = 28$ dollars more, for $8 - 4 = 4$ more sandwiches. So each sandwich adds $28 \div 4 = 7$ dollars, to the cost.

b. The delivery fee is 5 dollars. Sample reasoning: The first office paid $7 \cdot 4 = 28$ dollars for 4 sandwiches. The delivery fee is $33 - 28 = 5$ dollars.

