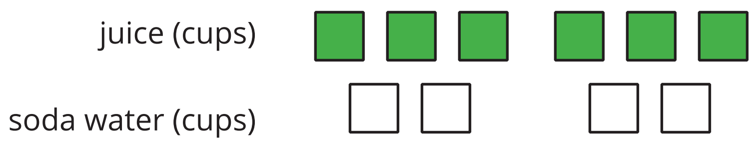
Unit 2 Family Support Materials

Ratios, Rates, and Percentages

Section A: What Are Ratios?

A **ratio** is an association between two or more quantities. For example, the cups of juice and the cups of soda water in a drink recipe form a ratio. Ratios can be represented with diagrams. Here is one diagram for a drink recipe:



Here are some correct ways to describe this diagram:

- The ratio of cups of juice to cups of soda water is $6 : 4$.
- The ratio of cups of soda water to cups of juice is 4 to 6.
- There are 6 cups of juice for every 4 cups of soda water.

We can also use other numbers to describe the quantities in this situation. For instance, we can say that there are 3 cups of juice for every 2 cups of soda water. The ratios $6 : 4$ and $3 : 2$ are **equivalent** because mixing juice and soda water in these amounts would make drinks that taste the same.

Two situations that can be described with **equivalent ratios** are the same in some important way. For example, mixing 1 ml of black paint and 10 ml of white paint would create the same shade of gray as mixing 3 ml of black paint and 30 ml of white paint, so these ratios of black paint to white paint are equivalent.

Here is a task to try with your student:

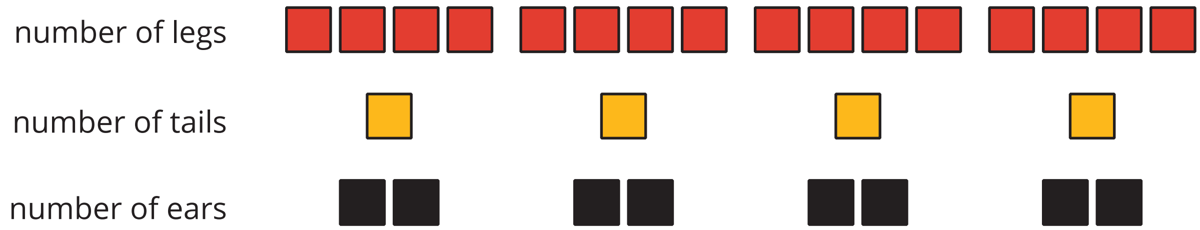
There are 4 horses in a stall. Each horse has 4 legs, 1 tail, and 2 ears.

1. Draw a diagram that shows the ratio of legs, tails, and ears in the stall.
2. Complete each statement.
    - The ratio of _______ to _______ to _______ is _______ : _______ : _______.
    - There are _______ ears for every tail. There are _______ legs for every ear.

Solution:

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

1. Sample response:



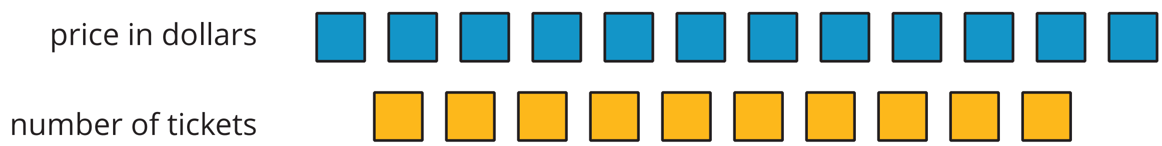
2. Sample response: The ratio of legs to tails to ears is $16 : 4 : 8$. There are 2 ears for every tail. There are 2 legs for every ear.

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

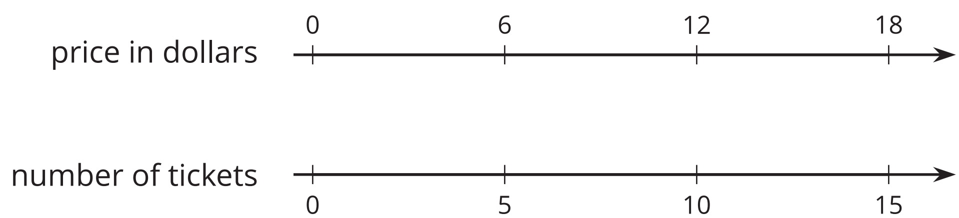
Section B: Representing Equivalent Ratios

There are different ways to represent equivalent ratios.

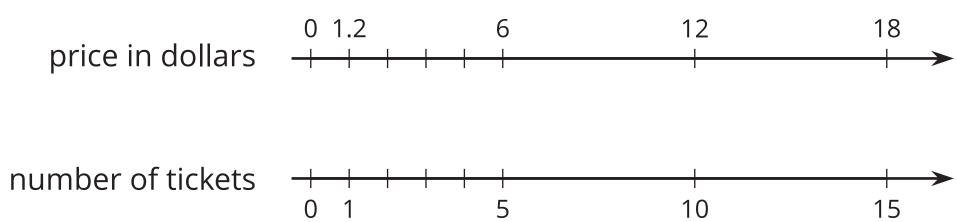
Let's say that the sixth grade class is selling raffle tickets at a price of $6 for 5 tickets. At that rate, 10 tickets cost $12. Some students may use diagrams with shapes to represent the situation. For example, here is a diagram representing 10 tickets for $12.



Drawing so many shapes becomes impractical. **Double number line diagrams** can be a quicker way to show equivalent ratios. Here is a diagram that represents the price in dollars for different numbers of raffle tickets all sold *at the same rate* of $6 for 5 tickets.



The diagram can be partitioned, extended, and marked up to find the prices for other numbers of tickets—including the price for 1 ticket, which is the **unit price**.



Double number line diagrams can be hard to use in problems with large amounts. If we tried to find the price of 300 tickets by extending the double number line diagram here, it would take 20 times more paper! A *table* is a better choice to represent this situation. Tables of equivalent ratios are useful because you can arrange the rows in any order. For example, a student may find the price for 300 raffle tickets by making the table shown.

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

price in dollars	number of tickets
6	5
1.20	1
360	300

Here is a task to try with your student:

At a constant speed, a train travels 45 miles in 60 minutes. At this rate, how far does the train travel in 12 minutes? If you get stuck, consider creating a table.

Solution:

9 miles. Sample reasoning:

time in minutes	distance in miles
60	45
1	0.75
12	9

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

Section D: Units of Measurement and Unit Conversion

In earlier grades, students converted yards to feet using the fact that 1 yard is 3 feet, and converted kilometers to meters using the fact that 1 kilometer is 1,000 meters. Now in grade 6, students convert units that do not always use whole numbers.

Students also use what they know about ratios and rates to reason about measurements in different units of measurement such as pounds and kilograms.

Suppose we weighed four objects in both pounds and kilograms and recorded the measurements in a table as shown here.

weight (pounds)	weight (kilograms)
22	10
88	40
33	15
40.7	18.5

The pair of values in each row forms a ratio, and the ratios in all rows of the table are equivalent. This understanding can help us convert between the two units of measurements.

Here is a task to try with your student:

Explain your strategy for each question.

1. Which is heavier, 1 pound or 1 kilogram?
2. A canoe weighs 99 pounds. How many kilograms does it weigh?
3. A watermelon weighs 12 kilograms. How many pounds does it weigh?

Solution:

Any correct strategy that your student understands and can explain is acceptable. Sample responses:

1. 1 kilogram is heavier than 1 pound. When we weigh the same object in pounds and kilograms, the number of pounds is more than the number of kilograms. It takes fewer kilograms to express the weight of the same object, so each kilogram must be heavier than each pound.

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

2. 45. Using the table, we can reason that 11 pounds is 5 kilograms. Multiplying each of these by 9 shows that 99 pounds is 45 kilograms.
3. 26.4. Using the table, we can find that each kilogram is equal to about 2.2 pounds. This means that if we know an object's weight in kilograms, we can multiply it by 2.2 to find its weight in pounds. $12 \cdot (2.2) = 26.4$

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

Section E: Rates

Who biked faster: Andre, who biked 25 miles in 2 hours, or Lin, who biked 30 miles in 3 hours?

One strategy would be to calculate a unit rate for each person. A **unit rate** is an equivalent ratio expressed as something "per 1." For example, Andre's rate could be written as "$12\frac{1}{2}$ miles in 1 hour" or "$12\frac{1}{2}$ miles *per hour*." Lin's rate could be written "10 miles *per hour*."

By finding the unit rates, we can compare the distance that each person went in 1 hour to see that Andre biked faster.

Every ratio has *two* unit rates. In this example, we could also compute *hours per mile*: how many hours it took each person to cover 1 mile. Although not every rate has a special name, rates in "miles per hour" are commonly called **speed** and rates in "hours per mile" are commonly called **pace**.

Andre:

distance (miles)	time (hours)
25	2
1	0.08
12.5	1

Lin:

distance (miles)	time (hours)
30	3
10	1
1	0.1

Here is a task to try with your student:

Dry dog food is sold in bulk: 4 pounds for $16.00.

1. At this rate, what is the cost *per pound* of dog food?
2. At this rate, what is the amount of dog food you can buy *per dollar*?

Solution:

1. $4.00 per pound. $16 \div 4 = 4$
2. We get $\frac{1}{4}$ or 0.25 of a pound per dollar because $4 \div 16 = 0.25$.

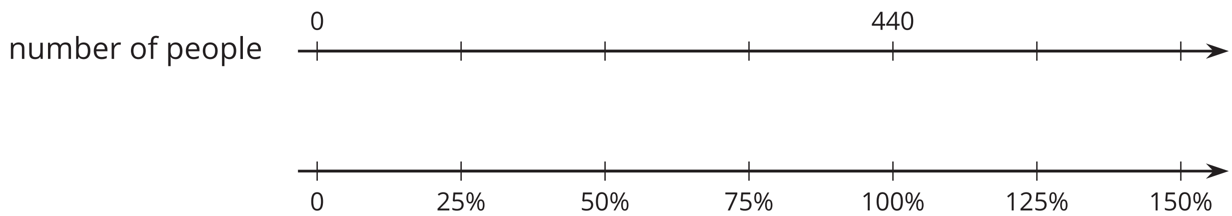
dog food (pounds)	cost (dollars)
4	16
1	4
0.25	1

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

Section F: Percentages

Let’s say 440 people attended a musical on its opening night. If 330 people were adults, what percentage of the attendees were adults? If attendance on the second night was 125% of the attendance on the opening night, how many people were there on the second night?

Students use their understanding of equivalent ratios and “rates per 1” to find **percentages**, which we can think of as “rates per 100.” They use double number line diagrams and tables to support their thinking and to answer questions such as the ones about attendance at a musical.



number of people	percentage
440	100%
110	25%
330	75%
550	125%

Toward the end of the unit, students develop more sophisticated strategies for finding percentages. For example, they can find 125% of 440 attendees by computing $\frac{125}{100} \cdot 440$. With practice, students will use these more efficient strategies and understand why they work.

Here is a task to try with your student:

For each question, explain your reasoning. If you get stuck, try creating a table or double number line for the situation.

1. A bottle of juice contains 16 ounces, and you drink 25% of the bottle. How many ounces did you drink?
2. You get 9 questions right in a trivia game, which is 75% of the questions. How many questions are in the game?
3. You planned to walk 8 miles, but you ended up walking 12 miles. What percentage of your planned distance did you walk?

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

Solution:

Any correct reasoning that a student understands and can explain is acceptable. Sample reasoning:

1. 4. 25% of the bottle is $\frac{1}{4}$ of the bottle, and $\frac{1}{4}$ of 16 is 4.
2. 12. If 9 questions is 75%, we can divide each by 3 to know that 3 questions is 25%. Multiplying each by 4 shows that 12 questions is 100%.
3. 150%. If 8 miles is 100%, then 4 miles is 50%, and 12 miles is 150%.

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.