Inequivalent Equations?

Let's see what happens when we square each side of an equation.

7.1 Notice and Wonder: 2 and -2

What do you notice? What do you wonder?

$x^2 = 4$

$(x - 2)(x + 2) = 0$

7.2 Careful When You Take the Square Root

Tyler was solving this equation:

$$x^2 - 1 = 3$$

He said, "I can add 1 to each side of the equation and it doesn't change the equation. I get $x^2 = 4$."

1. Priya said, "It does change the equation. It just doesn't change the solutions!" Then she showed these two graphs.

Figure A

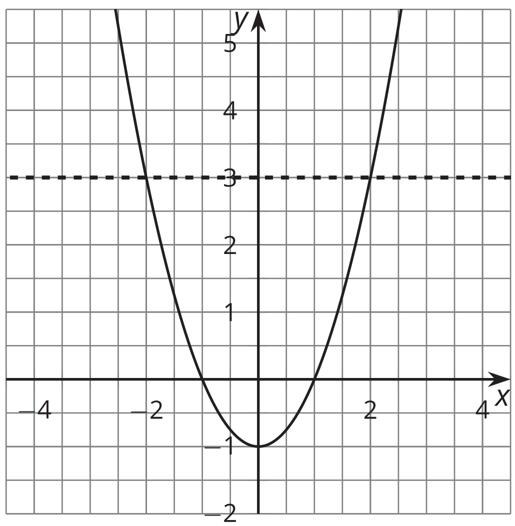
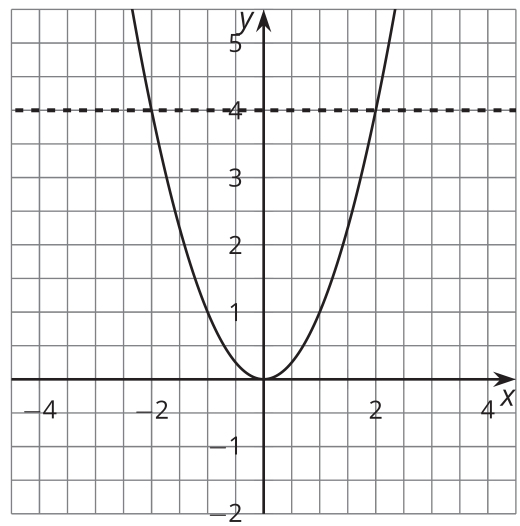


Figure B



   a. Figure A is related to the equation $x^2 - 1 = 3$ and Figure B is related to the equation $x^2 = 4$. What is graphed in each figure?

   b. How can you see the solution to the equation $x^2 - 1 = 3$ in Figure A?

   c. How can you see the solution to the equation $x^2 = 4$ in Figure B?

   d. Use the graphs to explain why the equations have the same solutions.

2. Tyler said, “Now I can take the square root of each side to get the solution to $x^2 = 4$. The square root of x^2 is x. The square root of 4 is 2.” He wrote:

$$x^2 = 4$$
$$\sqrt{x^2} = \sqrt{4}$$
$$x = 2$$

Priya said, “But the graphs show that there are *two* solutions!” What went wrong?

7.3 Another Way to Solve

Han was solving this equation:

$$\frac{x+3}{2} = 4$$

He said, "I know that half of $x+3$ is 4. So $x+3$ must be 8, since half of 8 is 4. This means that x is 5."

1. Use Han's reasoning to solve this equation: $(x+3)^2 = 4$.

2. What advice would you give to someone who was going to solve an equation like $(x+3)^2 = 4$?

7.4 What Happens When You Square Each Side?

Mai was solving this equation:

$$\sqrt{x-1} = 3$$

She said, "I can square each side of the equation to get another equation with the same solutions." Then she wrote:

$$\begin{aligned} \sqrt{x-1} &= 3 \\ (\sqrt{x-1})^2 &= 3^2 \\ x-1 &= 9 \\ x &= 10 \end{aligned}$$

1. Take turns with your partner explaining each step of Mai's work. Check to see if her solution makes the original equation true.

2. Tyler said, "I tried your technique to solve $\sqrt{x-1} = -3$, but it didn't work." Why doesn't it work? Explain or show your reasoning.

7.5 Solve These Equations with Square Roots in Them

Find the solution(s) to each of these equations, or explain why there is no solution. Be prepared to explain or show your reasoning.

1. $\sqrt{t+4}=3$

2. $-10=-\sqrt{a}$

3. $\sqrt{3-w}-4=0$

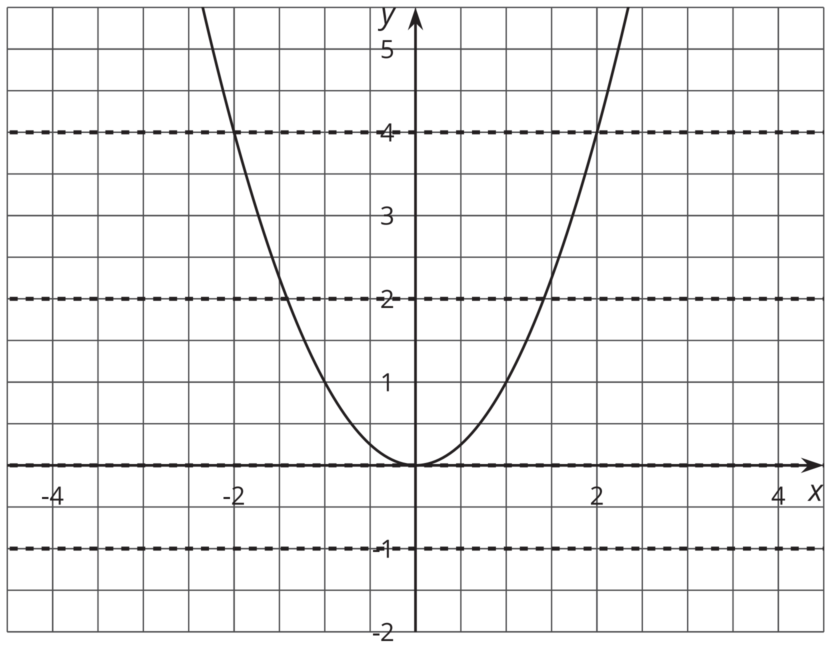
4. $\sqrt{z}+9=0$

Are you ready for more?

Are there values of a and b so that the equation $\sqrt{t+a}=b$ has more than one solution? Explain your reasoning.

Lesson 7 Summary

Every positive number has two square roots. You can see this by looking at the graph of $y = x^2$:



If y is a positive number like 4, then we can see that the line $y = 4$ crosses the graph in two places, so the equation $x^2 = 4$ will have two solutions, namely, $\sqrt{4}$ and -$\sqrt{4}$ (or 2 and -2). This is true for any positive number a: $y = a$ will cross the graph in two places, and $x^2 = a$ will have two solutions, $x = \sqrt{a}$ and $x = -\sqrt{a}$.

When we have a square root in an equation like $\sqrt{t} - 6 = 0$, we can isolate the square root and then square each side:

$$\begin{aligned} \sqrt{t} - 6 &= 0 \\ \sqrt{t} &= 6 \\ t &= 6^2 \\ t &= 36 \end{aligned}$$

But sometimes, squaring each side of an equation gives results that aren't solutions to the original equation. For example:

$$\begin{aligned} \sqrt{t} + 6 &= 0 \\ \sqrt{t} &= -6 \\ t &= (-6)^2 \\ t &= 36 \end{aligned}$$

Note that 36 is *not* a solution to the original equation, because $\sqrt{36} + 6$ doesn't equal 0. In fact, $\sqrt{t} + 6 = 0$ has no solutions, because it's impossible for the sum of two positive numbers to be zero.

Remember: Sometimes the new equation has solutions that the old equation doesn't have. Always check your solutions in the original equation!