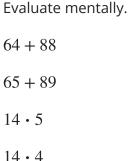


Lesson 21: Odd and Even Numbers

• Let's explore even and odd numbers.

21.1: Math Talk: Evens and Odds



21.2: Always Even, Never Odd

Here are some statements about the sums and products of numbers. For each statement:

- decide whether it is *always* true, true for *some* numbers but not others, or *never* true
- use examples to explain your reasoning
- 1. Sums:
 - a. The sum of 2 even numbers is even.
 - b. The sum of an even number and an odd number is odd.
 - c. The sum of 2 odd numbers is odd.
- 2. Products:
 - a. The product of 2 even numbers is even.
 - b. The product of an even number and an odd number is odd.



c. The product of 2 odd numbers is odd.

21.3: Even + Odd = Odd

How do we know that the sum of an even number and an odd number *must* be odd? Examine this proof and answer the questions throughout.

Let a represent an even number, b represent an odd number, and s represent the sum a+b.

1. What does it mean for a number to be even? Odd?

Assume that s is even, then we will look for a reason the original statement cannot be true. Since a and s are even, we can write them as 2 times an integer. Let a=2k and s=2m for some integers k and m.

2. Can this always be done? To convince yourself, write 4 different even numbers. What is the value for k for each of your numbers when you set them equal to 2k?

Then we know that a + b = s and 2k + b = 2m.

Divide both sides by 2 to get that $k + \frac{b}{2} = m$.

Rewrite the equation to get $\frac{b}{2} = m - k$.

Since m and k are integers, then $\frac{b}{2}$ must be an integer as well.

- 3. Is the difference of 2 integers always an integer? Select 4 pairs of integers and subtract them to convince yourself that their difference is always an integer.
- 4. What does the equation $\frac{b}{2} = m k$ tell us about $\frac{b}{2}$? What does that mean about b?
- 5. Look back at the original description of *b*. What is wrong with what we have discovered?

The logic for everything in the proof works, so the only thing that could've gone wrong was our assumption that s is even. Therefore, s must be odd.