Polynomial Division (Part 1)

Let's learn a way to divide polynomials.

12.1 Notice and Wonder: A Different Use for Diagrams

What do you notice? What do you wonder?

A. $(x-3)(x+5)=x^2+2x-15$

	x	5
x	x^2	$5x$
-3	$-3x$	-15

B. $(x-1)(x^2+3x-4)=x^3+2x^2-7x+4$

	x^2	$3x$	-4
x	x^3	$3x^2$	$-4x$
-1	$-x^2$	$-3x$	+4

C. $(x-2)(?)=(x^3-x^2-4x+4)$

x	x^3		
-2			

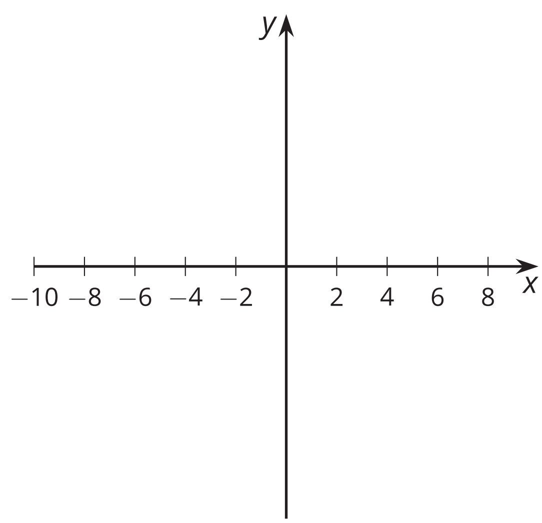
12.2 Factoring with Diagrams

Priya wants to sketch a graph of the polynomial f defined by $f(x) = x^3 + 5x^2 + 2x - 8$.

She knows $f(1) = 0$, so she suspects that $(x - 1)$ could be a factor of $x^3 + 5x^2 + 2x - 8$ and writes $(x^3 + 5x^2 + 2x - 8) = (x - 1)(?x^2 + ?x + ?)$ and draws a diagram.

x	x^3		
-1			

1. Finish Priya's diagram.
2. Write $f(x)$ as the product of $(x - 1)$ and another factor.
3. Write $f(x)$ as the product of three linear factors.
4. Draw a sketch of $y = f(x)$.



12.3 More Factoring with Diagrams

Here are some polynomial functions with one or more known factors. Rewrite each polynomial as a product of linear factors.

Note: you may not need to use all the columns in each diagram. For some problems, you may need to make another diagram.

1. $A(x) = x^3 - 7x^2 - 16x + 112$, $(x - 7)$

	x^2				
x	x^3	0			
-7	$-7x^2$				

2. $B(x) = 2x^3 - x^2 - 27x + 36$, $\left(x - \frac{3}{2}\right)$

	$2x^2$				
x	$2x^3$	$2x^2$			
$-\frac{3}{2}$	$-3x^2$				

3. $C(x) = x^3 - 3x^2 - 13x + 15$, $(x + 3)$

x					
3					

4. $D(x) = x^4 - 13x^2 + 36$, $(x - 2)$, $(x + 2)$

(Hint: $x^4 - 13x^2 + 36 = x^4 + 0x^3 - 13x^2 + 0x + 36$)

5. $F(x) = 4x^4 - 15x^3 - 48x^2 + 109x + 30$, $(x - 5)$, $(x - 2)$, $(x + 3)$

Are you ready for more?

A diagram can also be used to divide polynomials even when a factor is not linear. Suppose we know $(x^2 - 2x + 5)$ is a factor of $x^4 + x^3 - 5x^2 + 23x - 20$. We could write $(x^4 + x^3 - 5x^2 + 23x - 20) = (x^2 - 2x + 5)(?x^2 + ?x + ?)$. Make a diagram, and find the missing factor.

Lesson 12 Summary

What are some things that could be true about the polynomial function defined by $p(x) = x^3 - 5x^2 - 2x + 24$ if we know $p(\text{-}2) = 0$?

- Thinking about the graph of the polynomial, the point $(\text{-}2, 0)$ must be on the graph as a horizontal intercept.
- Thinking about the expression written in factored form, $(x + 2)$ *could* be one of the factors, since $x + 2 = 0$ when $x = \text{-}2$.

How can we figure out whether $(x + 2)$ actually is a factor?

If we assume that $(x + 2)$ is a factor, then there is some other polynomial $q(x) = ax^2 + bx + c$ where a, b, and c are real numbers and $p(x) = (x + 2)q(x)$. In the past we have expanded $(x + 2)(ax^2 + bx + c)$ to find $p(x) = (x + 2)q(x)$. Instead, we can work out the values of a, b, and c by thinking through the calculation backward.

One way to organize our thinking is to use a diagram. First, fill in $(x + 2)$ and the leading term of $p(x)$, x^3. From this we can see the leading term of $q(x)$ must be x^2, meaning $a = 1$, since $x \cdot x^2 = x^3$.

	x^2		
x	x^3		
$+2$			

We can fill in the rest of the diagram using similar thinking and paying close attention to the signs of each term. For example, we put in a $2x^2$ in the bottom left cell because that's the product of 2 and x^2. But that means we need to have a $\text{-}7x^2$ in the middle cell of the middle row, since that's the only other place we will get an x^2 term, and we need to get $\text{-}5x^2$ once all the terms are collected. Continuing in this way, we get the completed table:

	x^2	$\text{-}7x$	$+12$
x	x^3	$\text{-}7x^2$	$+12x$
$+2$	$+2x^2$	$\text{-}14x$	$+24$

Collecting all the terms in the interior of the diagram, we see that $x^3 - 5x^2 - 2x + 24 = (x + 2)(x^2 - 7x + 12)$, so $q(x) = x^2 - 7x + 12$. Notice that the 24 in the bottom right was exactly what we needed, and it is how we know that $(x + 2)$ is a factor of $p(x)$. With a bit more factoring, we can say that $p(x) = (x + 2)(x - 3)(x - 4)$.