Lots of Lines (Part 1)

Narrator: Diego, Jada, and Noah were given the following task: "Prove that if a point C is the same distance from A as it is from B, then C must be on the perpendicular bisector of AB." At first they were really stuck.

Noah: How do you prove a point is on a line?

Narrator: Their teacher gave them the hint, "Another way to think about it is to draw a line that you know C is on, and prove that line has to be the perpendicular bisector." They each drew a line and thought about their pictures.

Diego: I drew a line through C that was perpendicular to AB and through the midpoint of AB. That line is the perpendicular bisector of AB and C is on it, so that proves C is on the perpendicular bisector."

Jada: I thought the line through C would probably go through the midpoint of AB so I drew that and labeled the midpoint D. Triangle ACB is isosceles, so angles A and B are congruent, and AC and BC are congruent. And AD and DB are congruent because D is a midpoint. That made two congruent triangles by the Side-Angle-Side Triangle Congruence Theorem. So I know angle ADC and angle BDC are congruent, but I still don't know if DC is the perpendicular bisector of AB.

Noah: In the Isosceles Triangle Theorem proof, Mai and Kiran drew an angle bisector in their isosceles triangle, so I'll try that. I'll draw the angle bisector of angle ACB. The point where the angle bisector hits AB will be D. So triangles ACD and BCD are congruent, which means AD and BD are congruent, so D is a midpoint and CD is the perpendicular bisector.

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