A Radical Identity

Let's use an identity with square roots.

13.1 Quadratic Zeros

1. Let $f(x) = (x - 3)(x - 4)$.
   a. For what values of x is the function f equal to 0? Recall that these values are called *zeros*.
   b. Rewrite the equation in standard form, $f(x) = ax^2 + bx + c$.
   c. For this function, how are b and c related to the zeros?
   d. Is this the only quadratic function with zeros at 3 and 4?
2. Let $g(x) = (x - s)(x - t)$.
   a. What are the zeros of g?
   b. Rewrite the equation in standard form.

13.2 An Identity from Zeros

Let s and t be the two solutions from the quadratic formula.

$$s = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad t = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

1. Multiply s and t to find a simple expression in terms of a, b, and c.
2. Add s and t to find a simple expression in terms of a, b, and c.
3. Show that $a(x - s)(x - t) = ax^2 + bx + c$.
4. Use the connection between the variables s, t, b, and c to write the quadratic function $f(x)$ in standard form that has zeros at $x = 1 + \sqrt{5}$ and $x = 1 - \sqrt{5}$ and a quadratic coefficient of 1 ($a = 1$). Explain or show your reasoning.

Are you ready for more?

What about complex solutions? Let $s = m + ni$ and $t = m - ni$ for real numbers m and n and imaginary number i.

Write a function in standard form that represents all quadratic functions with zeros at s and t in terms of m and n.

13.3 Removing Square Roots

1. Finish this identity: $d \cdot 1 =$ ________. Are there any restrictions on what d can be for the identity to be true?
2. Finish this identity: $\frac{c}{c} =$ ________. Are there any restrictions on what c can be for the identity to be true?

   Pause here for a discussion.
3. Show that $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$
4. Show that $\frac{2}{\sqrt{5}+\sqrt{3}} = \sqrt{5} - \sqrt{3}$

Lesson 13 Summary

Mathematical identities can be useful in a variety of situations.

In an earlier course, we learned how to derive the quadratic formula using the method of completing the square on a quadratic equation. We can check the formula by reconstructing the standard form of a quadratic equation using the zeros guaranteed by the quadratic formula. We can start with writing the factored form using the zeros as

$$a\left(x - \frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(x - \frac{-b-\sqrt{b^2-4ac}}{2a}\right) = 0.$$

After distributing the left side of the equation, it nicely becomes $ax^2 + bx + c = 0$.

Identities can also be used to rewrite expressions to be in a form that is clearer to understand. For example, the value of the fraction $\frac{5}{\sqrt{7}+\sqrt{2}}$ is not very easy to estimate without a calculator. Let's rewrite the expression using two identities: the difference of squares identity, $(a + b)(a - b) = a^2 - b^2$, and the identity $\frac{c}{c} = 1$ when $c \neq 0$.

$$\begin{aligned}\frac{5}{\sqrt{7}+\sqrt{2}} &= \frac{5}{\sqrt{7}+\sqrt{2}} \cdot \frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}-\sqrt{2}} \\ &= \frac{5(\sqrt{7}-\sqrt{2})}{(\sqrt{7})^2-(\sqrt{2})^2} \\ &= \frac{5(\sqrt{7}-\sqrt{2})}{7-2} \\ &= \sqrt{7} - \sqrt{2}\end{aligned}$$

In this form, we can estimate that $\sqrt{7}$ is approximately 2.5 and $\sqrt{2}$ is a little more than 1, so the expression has a value of approximately 1.25. Checking on a calculator, we can see that the original fraction and the final expression are equal and have a value of approximately 1.232, which is close to our estimation.

