Unknown Exponents

Let’s find unknown exponents.

8.1 A Bunch of x’s

Solve each equation. Be prepared to explain your reasoning.

1. $\frac{x}{3} = 12$

2. $3x^2 = 12$

3. $x^3 = 12$

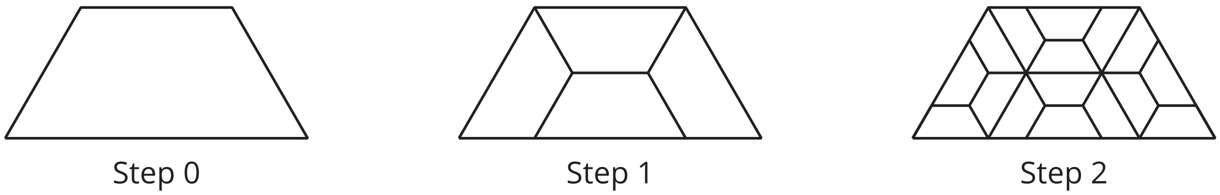
4. $\sqrt[3]{x} = 12$

5. $\sqrt{3x} = 12$

6. $\frac{3}{x} = 12$

8.2 A Tessellated Trapezoid

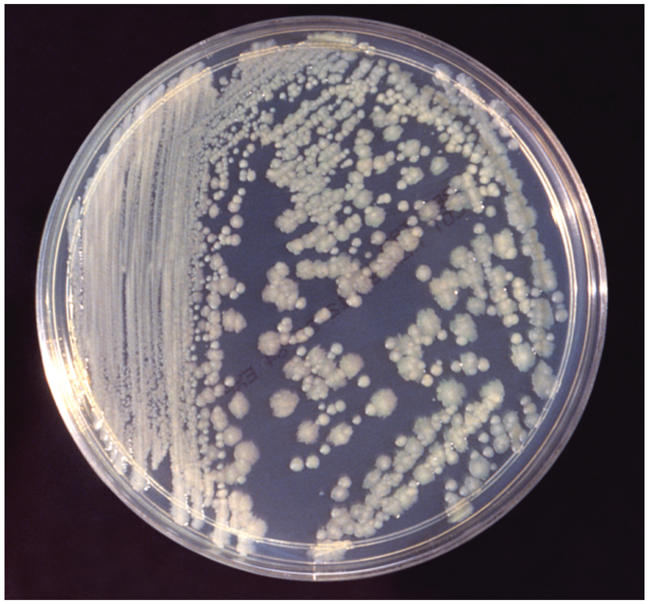
Here is a pattern showing a trapezoid decomposed into similar trapezoids at each step.



1. If n is the step number, how many of the smallest trapezoids are there when n is 4? What about when n is 7?

2. At a certain step, k, there are 262,144 smallest trapezoids.
   a. Write an equation to represent the relationship between k and the number of trapezoids in step k.
   b. Explain to a partner how you might find the value of k.

8.3 Successive Splitting



In a lab, a colony of 100 thousand bacteria is placed on a petri dish. The population grows exponentially, tripling every hour.

1. How would you estimate or find the population of bacteria in:
   a. 4 hours?
   b. 90 minutes?
   c. $\frac{1}{2}$ hour?
2. How would you estimate or find the number of hours it would take the population to grow to:
   a. 1,000 thousand bacteria?
   b. double the initial population?

Are you ready for more?

A $1,000 investment increases in value by 5% each year. About how many years does it take for the value of the investment to double? Explain how you know.

8.4 Missing Values

Complete the tables.

x			-1	0	$\frac{1}{2}$	1			5		
2^x	$\frac{1}{32}$	$\frac{1}{4}$	$\frac{1}{2}$				4	16		256	1,024

x				$\frac{1}{3}$	$\frac{1}{2}$				
5^x	$\frac{1}{25}$	$\frac{1}{5}$	1			5	125	625	3,125

Be prepared to explain how you found the missing values.

Lesson 8 Summary

Sometimes we know the value of an exponential expression but we don't know the exponent that produces that value.

For example, suppose the population of a town was 1 thousand. Since then, the population has doubled every decade and is currently at 32 thousand. How many decades has it been since the population was 1 thousand?

If we say that d is the number of decades since the population was 1 thousand, then $1 \cdot 2^d$, or just 2^d, represents the population, in thousands, after d decades. To answer the question, we need to find the exponent in $2^d = 32$. We can reason that since $2^5 = 32$, it has been 5 decades since the population was 1 thousand people.

When did the town have 250 people? Assuming that the doubling started before the population was measured to be 1 thousand, we can write: $2^d = 0.25$, or $2^d = \frac{1}{4}$. We know that $2^{-2} = \frac{1}{4}$, so the exponent d has a value of -2. The population was 250 two decades before it was 1,000.

But it may not always be so straightforward to calculate. For example, it is harder to tell the value of d in $2^d = 805$ or in $2^d = 4.5$. In upcoming lessons, we'll learn more ways to find unknown exponents.