Unit 4, Lesson 6

Squares and Square Roots

Goals

- Comprehend that the symbol $\sqrt{x}$ denotes only the positive square root of x.
- Explain why equations of the form $x^2 = a$ can have two solutions, while equations of the form $\sqrt{x} = a$ cannot have more than one solution.

Learning Targets

- I understand that the square root symbol means the positive square root.

Lesson Narrative

The purpose of this lesson is for students to reason from the meaning of a square and the meaning of a square root to solve equations of the form $x^2 = a$ and $\sqrt{x} = a$, where a is positive. They use graphs to understand that positive numbers always have two square roots, one positive and one negative. They also consider why we might want $\sqrt{x}$ to represent a single value rather than both values that square to make x. Then they use graphs to understand why equations like $x = \sqrt{5}$ have only one solution. There is no focus on equation-solving procedures in this lesson, which is something students will turn their attention to and develop techniques regarding in later lessons.

Note that there are certain claims like, "The equation $x^2 = -1$ has no solution," which would be more precisely stated as "The equation $x^2 = -1$ has no *real* solution." However, students don't know about any numbers other than real numbers, so it does not make sense to make this distinction at this time.

Students attend to precision when they reason about solutions to equations involving squares and square roots from the meaning of the $\sqrt{}$ symbol (MP6).

Standards

Addressing	**HSA-REI.A.2, HSF-IF.C.7.b**
Building Toward	**HSF-IF.C.7.b**

Instructional Routines

- Math Talk
- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Math Community Chart: Cool-down

Student Facing Learning Goals

Let's compare equations with squares and square roots.

6.1 Math Talk: Four Squares

5 min

Warm-up

Activity Narrative

This *Math Talk* focuses on finding solutions to equations of the form $x^2 = a$. It encourages students to think about square roots and to rely on what they know about numbers to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students examine roots more closely. To find all solutions, students need to look for and make use of structure (MP7).

Standards

Addressing **HSA-REI.A.2**

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Find the solutions of each equation mentally.

- $x^2 = 4$
- $x^2 = 2$
- $x^2 = 0$
- $x^2 = -1$

Student Response

- x is either 2 or -2. Sample reasoning: $2^2 = 4$ and $(-2)^2 = 4$.
- x is either $\sqrt{2}$ or -$\sqrt{2}$. Sample reasoning: A number that squares to make 2 is $\sqrt{2}$. Its opposite also squares to make 2.
- $x = 0$. Sample reasoning: $0^2 = 0$ and it does not have an opposite, so it is the only solution.

- This equation has no solution. Sample reasoning: Multiplying a number by itself is always positive, so there are no numbers that can solve this equation.

Activity Synthesis

The goal of this discussion is to review students' strategies for reasoning about solutions to equations of the form $x^2 = a$.

To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"
- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. Examples:

- "First, I ________ because ________."
- "I noticed ________, so I ________."

Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

6.2 Finding Square Roots

15 min

Activity Narrative

The purpose of this activity is for students to consider some reasons for the convention that $\sqrt{x}$ means the positive square root of x. Students think about what it would mean if the square root function were not a function. In the synthesis, the convention that makes $\sqrt{x}$ a function is introduced. It should be emphasized that there are still two square roots of every positive number, but the square root function gives us only one of those. Students will explore the consequences of this convention in future activities.

Standards

Building Toward **HSF-IF.C.7.b**

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Display this equation:

$\sqrt{4} + \sqrt{9} = ?$

Tell students that in this activity, they will think about the answer to this question. Give students 2 minutes to read the statement, think, and write down their answers individually, and another 2 minutes for pairs to share their thoughts. Follow with a whole-class discussion.

Access for English Language Learners

MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to Clare's question. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Student Task Statement

Clare was adding $\sqrt{4}$ and $\sqrt{9}$, and at first she wrote $\sqrt{4} + \sqrt{9} = 2 + 3$. But then she remembered that 2 and -2 both square to make 4, and that 3 and -3 both square to make 9.

She wrote down all the possible combinations:

2 + 3 = 5
2 + (-3) = -1
(-2) + 3 = 1
(-2) + (-3) = -5

Then she wondered, "Which of these are the same as $\sqrt{4} + \sqrt{9}$? All of them? Or only some? Or just one?"

How would you answer Clare's question? Give reasons that support your answer.

Student Response

Sample responses:

- I think only one of these could be the answer, because there can only be one result when we add numbers.
- I think all of these are right, because positive numbers have two square roots and these are all the possibilities.

Are You Ready for More?

1. How many solutions are there to each equation?
    a. $x^3 = 8$
    b. $y^3 = -1$
    c. $z^4 = 16$
    d. $w^4 = -81$
2. Write a rule to determine how many solutions there are to the equation $x^n = m$ where n and m are non-zero integers.

Extension Student Response

1. a. 1

b. 1

c. 2

d. 0

2. Sample response: If n is odd, there is 1 solution. If n is even, there are 2 solutions when m is positive, and 0 solutions if m is negative.

Activity Synthesis

Invite students to share their answers and reasons. The goal of the discussion is for students to consider some reasons why we might want the operation of taking the square root to give us only one number. Here are some questions for discussion if needed:

- "The solutions to the equation $x^2 = 2$ are approximately 1.414 and -1.414. If $\sqrt{2}$ represents only one of the exact solutions, which of the approximations would you choose? How could you write the other exact solution?" (I would choose the positive number because $\sqrt{2}$ looks positive to me. The other solution could be written -$\sqrt{2}$.)
- "Suppose $\sqrt{2}$ represents both solutions to $x^2 = 2$. Now suppose that Clare is solving this equation because it represents a square with an area of 2 square units and she wants to find the side length of the square. How could she show that she wants only the positive solution?" (She could write out the phrase "the positive part of $\sqrt{2}$.")
- "Do you think it makes sense to choose the meaning of $\sqrt{2}$? That is, would it make sense to let that symbol sometimes mean only the positive number and sometimes mean both numbers?" (That would get very confusing. I think it makes sense to choose one of the meanings and stick with it.)
- "Would you rather the symbol $\sqrt{2}$ represent only one of the solutions to $x^2 = 2$ or both solutions?" (I would rather it represent only the positive solution.)
- Tell students that mathematicians use the convention that $\sqrt{a}$, or $a^{\frac{1}{2}}$, represents only the positive solution to $x^2 = a$. This convention helps in being precise about what is meant.

15 min

6.3 One Solution or Two?

Activity Narrative

In this activity, students use a table of values to draw the graph of $y = \sqrt{x}$ and consider whether it is a function. Then students use the graph to reinforce the idea that an equation like $\sqrt{x} = 4$ has one solution while $x^2 = 4$ has two solutions.

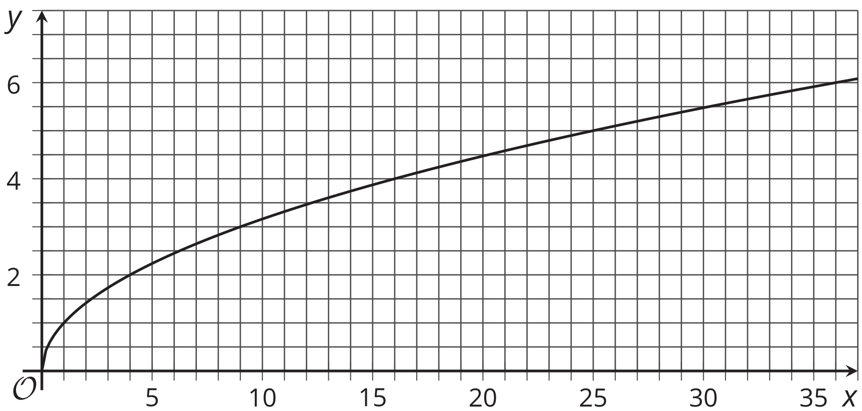
Standards

Addressing **HSF-IF.C.7.b**

Launch

Arrange students in groups of 2. Ask students if they recall the meaning of the term "function" and to define it. A *function* is a rule that takes inputs from one set and assigns them to outputs from another set. Each input is assigned exactly one output.

Allow students 2–3 minutes to complete the table and sketch the graph, and then pause the class to reveal a more accurate graph of $y = \sqrt{x}$.



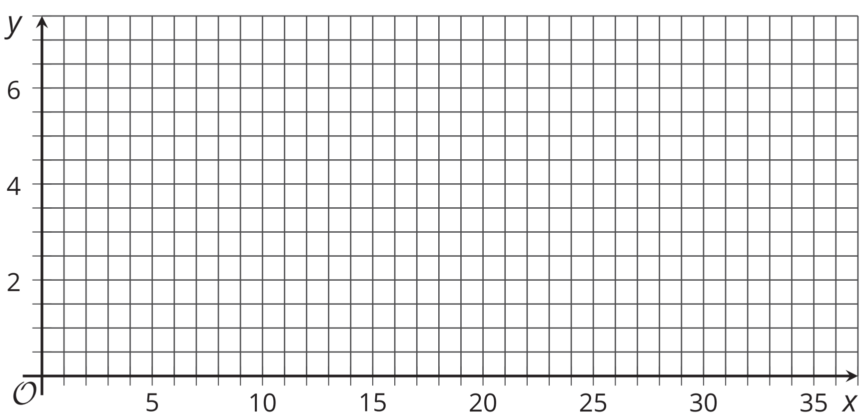
If this graph is recreated using technology, ensure that the y-axis grid is in steps of 0.5 and the x-axis extends to at least 36.

Student Task Statement

1. Complete the table.

x	0	1	4	9	16	25	36
$\sqrt{x}$							

2. Use the values from the table to plot seven points on the graph of $y = \sqrt{x}$. Then sketch the graph by smoothly connecting the points you drew.

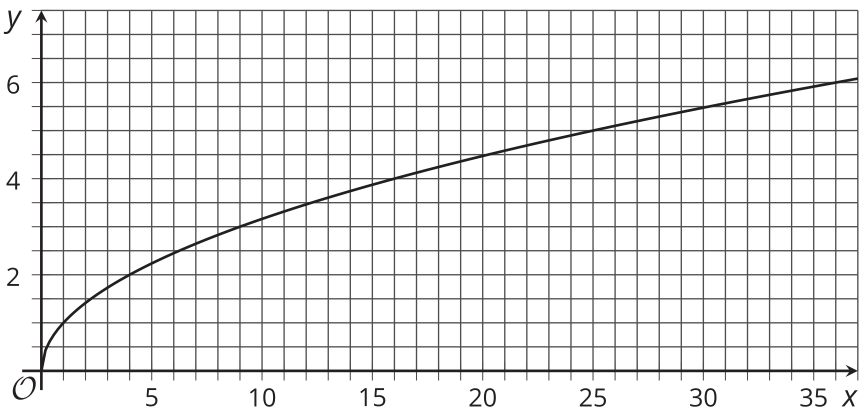


3. Is the rule $y = \sqrt{x}$ a function? Explain your reasoning.
4. Explain how you could use the graph to find any solutions to the equation $\sqrt{x} = 5$. How many solutions are there?
5. Use the graph to approximate the value of $\sqrt{5}$. Explain your reasoning.
6. Approximate any solutions to the equation $x^2 = 20$. Explain your reasoning.

Student Response

1. Complete the table.

x	0	1	4	9	16	25	36
$\sqrt{x}$	0	1	2	3	4	5	6

2. 

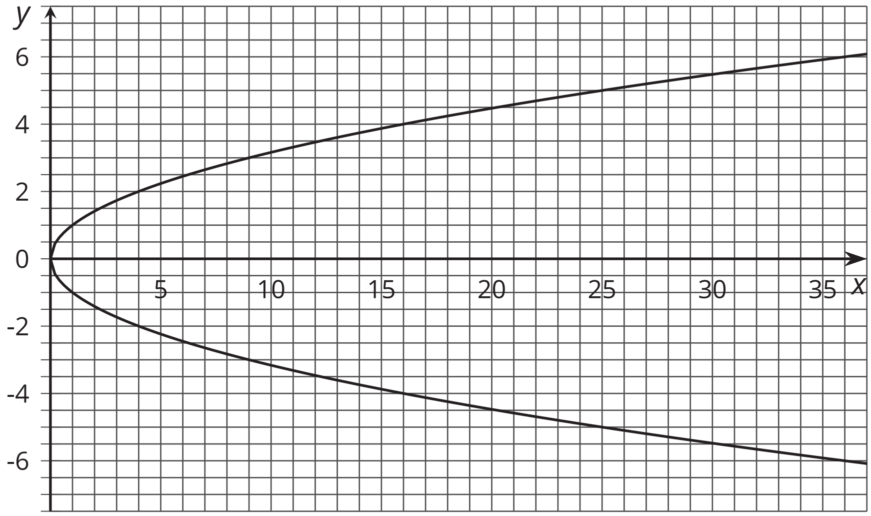
3. It is a function. Sample reasoning: Each input has exactly one output because each value on the x-axis has only one part of the curve above it.
4. Sample response: Draw the line $y = 5$ and find where it intersects the graph of $y = \sqrt{x}$. There is one solution, 25.
5. Sample response: About 2.25. Find 5 on the x-axis, then follow the grid line straight up to find the point where it hits the graph. Use the horizontal grid lines to find an approximate y-value for this point.
6. Sample response: About 4.5 and -4.5. The solutions should be $\sqrt{20}$ and -$\sqrt{20}$. I used the graph to approximate this value by finding a point on the graph of $y = \sqrt{x}$ at around $(20, 4.5)$.

Activity Synthesis

The purpose of this discussion is to emphasize that the rule $y = \sqrt{x}$ is a function. Invite students to share their reasoning about why it is a function. Then ask students,

- “Why is it important that the rule $y = \sqrt{x}$ is a function when you want to use the graph of $y = \sqrt{x}$ to find the value of $\sqrt{5}$? (Because it is a function, there is only one value on the graph above or below 5 on the x-axis.)
- “Describe what a graph of the equation $y^2 = x$ looks like.” (It would look like $y = \sqrt{x}$, but also have another part in which the shape is reflected over the x-axis. It looks like a parabola opening to the right.)

Display the graph of $y^2 = x$.



Then ask students, "How can you tell that the graph of $y^2 = x$ is not a function?" (For some inputs it has more than one output. For example, the points $(4, 2)$ and $(4, -2)$ are both on the graph.)

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, to show that it is a function and to estimate the value of $\sqrt{5}$, draw a vertical line at $x = 5$, showing how it intersects the graph only once.
Supports accessibility for: Visual-Spatial Processing

Lesson Synthesis

The purpose of the discussion is for students to recognize that $\sqrt{a}$ is a single, positive value (when a is positive), but that there are two square roots of a as solutions to the equation $x^2 = a$. Here are some questions for discussion:

- "What do you know about the number $\sqrt{17}$?" (It is a number that squares to make 17. It is positive. It is a little greater than 4.)
- "What are all the numbers that square to make 17? In other words, what are the square roots of 17?" ($\sqrt{17}$ and $-\sqrt{17}$)
- "Let's say that a is a positive number. What do you know about the number $\sqrt{a}$?" (It is a number that squares to make a. It is positive.)
- "What are all the numbers that square to make a? That is, what are the square roots of a?" ($\sqrt{a}$ and $-\sqrt{a}$)
- "Is it possible for $\sqrt{a}$ to be equal to -10? Explain your reasoning." (No, it's not possible because $\sqrt{a}$ is a positive number and -10 is not.)

6.4 Squares and Roots

Cool-down

5 min

Standards

Addressing **HSA-REI.A.2**

Launch

It is not expected that students have a procedure for answering the second question other than by inspection. They will have more opportunity to reason about strategies for answering questions like it in the next lesson.

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

- "What norm(s) should stay the way they are?"
- "What norm(s) do you think should be made more clear? How?"
- "What norms are missing that you would add?"
- "What norm(s) should be removed?"

Tell students to first complete the *Cool-down* and then, on the same sheet, to record their answers to one or more of the Math Community questions.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Plan to discuss the potential revisions over the next few lessons.

Student Task Statement

1. Find the exact solution(s) to each equation.
   a. $x^2 = 7$
   b. $\sqrt{y} = 7$
2. How would you explain that the equation $\sqrt{z} = -7$ does not have a solution?

Student Response

1.
   a. $x = \sqrt{7}$ or $x = -\sqrt{7}$
   b. $y = 49$
2. Sample response: $\sqrt{z} = -7$ does not have a solution because the left side is a positive number and the right side is a negative number. A positive can't equal a negative.

Responding to Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 6 Summary

To avoid confusion, we use the convention that $\sqrt{a}$ represents a single positive number (when a is positive). This allows us to easily describe both solutions to the equation $x^2 = a$. The solutions are $\sqrt{a}$ and $-\sqrt{a}$.

The equation $x^2 = 11$ has two solutions, because $\sqrt{11}^2 = 11$, and also $\left(-\sqrt{11}\right)^2 = 11$.

The equation $\sqrt{x} = 11$ only has one solution, namely 121.

The equation $\sqrt{x} = \sqrt{11}$ only has one solution, namely 11.

The equation $\sqrt{x} = -11$ doesn't have any solutions, because the left side is positive and the right side is negative, which is impossible, because a positive number cannot equal a negative number.

Lesson 6 Practice Problems

1 Student Task Statement

Select **all** solutions to the equation $x^2 = 7$.

A. $\sqrt{7}$

B. $-\sqrt{7}$

C. 49

D. -49

Solution

A, B

2 Student Task Statement

Find the solution(s) to each equation, if there are any.

a. $x^2 = 9$
b. $\sqrt{x} = 3$
c. $\sqrt{x} = -3$

Solution

a. 3 or -3
b. 9
c. No solution, because $\sqrt{x}$ is defined to be positive.

3 Student Task Statement

a. If c is a positive number, how many solutions does $x^2 = c$ have? Explain your reasoning.
b. If c is a positive number, how many solutions does $\sqrt{x} = c$ have? Explain your reasoning.

Solution

a. Two solutions. Sample reasoning: Every positive number has two numbers you can square to get the number back. For instance, $8^2 = 64$, and $(-8)^2 = 64$.
b. One solution. Sample reasoning: Say that $c = 4$. The only number you can take the square root of and get 4 is 16. Negative 16 doesn't work.

4 from Unit 4, Lesson 3

Student Task Statement

Suppose that a friend missed class and never learned what $37^{\frac{1}{3}}$ means.

a. Use exponent rules that your friend would already know to show your friend how to calculate $(37^{\frac{1}{3}})^3$.

b. Explain why this means that $37^{\frac{1}{3}}$ is the cube root of 37.

Solution

a. $(37^{\frac{1}{3}})^3 = 37^{\frac{1}{3} \cdot 3} = 37^1 = 37$

b. Because $37^{\frac{1}{3}}$ cubed equals 37, then $37^{\frac{1}{3}}$ must be the cube root of 37.

5 from Unit 4, Lesson 5

Student Task Statement

Write each expression without using exponents.

a. $5^{\frac{2}{3}}$

b. $4^{-\frac{3}{2}}$

Solution

a. $\sqrt[3]{25}$

b. $\frac{1}{8}$