Unit 3, Lesson 10

Distinguishing Circumference and Area

Let's contrast circumference and area.

10.1 A Plate of Marbles

About how many marbles can fit on the plate in a single layer? Be prepared to explain your reasoning.



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10.2 Card Sort: Circle Problems

Your teacher will give you a set of cards with questions about circles.

1. Take turns with your partner to sort the cards into two groups based on whether you would use the circumference or the area of the circle to answer the question
   a. For each card that your partner sorts, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
   b. For each card that you sort, explain to your partner how you know which group it goes in.
2. Your teacher will assign you one card to examine more closely. What additional information would you need in order to answer the question on your card?

3. Estimate measurements for the circle that is described on your card.

4. Use your estimates to calculate the answer to the question.

10.3 Visual Display of Circle Problem

In the previous activity you estimated the answer to a question about circles.

Create a visual display that includes:

- The question you were answering.
- A diagram of a circle labeled with your estimated measurements.
- Your thinking, organized so that others can follow it.
- Your answer, expressed in terms of π and also expressed as a decimal approximation.

10.4 Analyzing Circle Claims

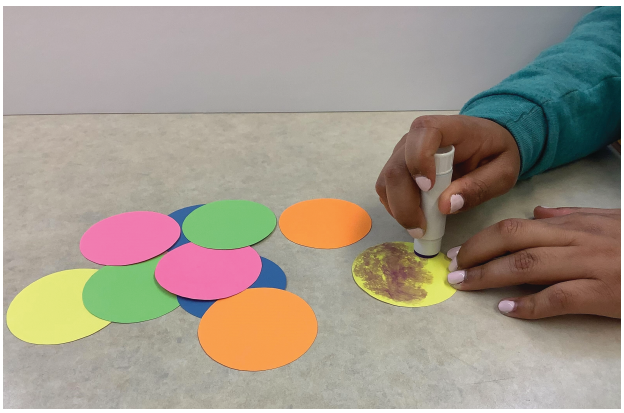
Here are two students' answers for each question. Do you agree with either of them? Explain or show your reasoning.

1. How many feet are traveled by a person riding once around the merry-go-round?



- Clare says, "The radius of the merry-go-round is about 4 feet, so the distance around the edge is about 8π feet."
- Andre says, "The diameter of the merry-go-round is about 4 feet, so the distance around the edge is about 4π feet."

2. How much room is there to put glue on the back of a paper circle?



- Clare says "The radius of the circle is about 3 centimeters, so the space for glue is about 6π cm^2."
- Andre says "The diameter of the circle is about 3 inches, so the space for glue is about 2.25π in^2."

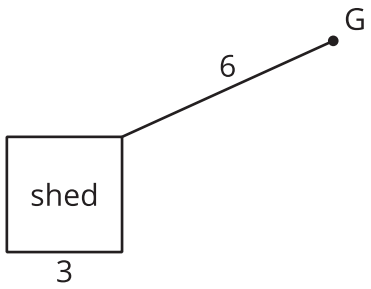
3. How far does the unicycle move when the wheel makes 5 full rotations?



- Clare says, "The diameter of the unicycle wheel is about 0.5 meters. In 5 complete rotations it will go about $\frac{5}{2}\pi$ m^2."
- Andre says, "I agree with Clare's estimate of the diameter, but that means the unicycle will go about $\frac{5}{4}\pi$ m."

Are you ready for more?

A goat (point G) is tied with a 6-foot rope to the corner of a shed. The floor of the shed is a square whose sides are each 3 feet long. The shed is closed and the goat can't go inside. The space all around the shed is flat and grassy, and the goat can't reach any other structures or objects. What is the area over which the goat can roam?



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10.5 Info Gap: Merry-go-round and Unicycle

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because . . . ."

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning, and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Lesson 10 Summary

Sometimes we need to find the circumference of a circle, and sometimes we need to find the area.

Here are some examples of quantities related to the circumference of a circle:

- The length of a circular path.
- The distance a wheel will travel after one complete rotation.
- The length of a piece of rope coiled in a circle.

Here are some examples of quantities related to the area of a circle:

- The amount of land that is cultivated on a circular field.
- The amount of frosting needed to cover the top of a round cake.
- The number of tiles needed to cover a round table.

In both cases, the radius (or diameter) of the circle is all that is needed to make the calculation. The circumference of a circle with radius r is $2\pi r$ while its area is πr^2. The circumference is measured in linear units (such as cm, in, km) while the area is measured in square units (such as cm^2, in^2, km^2).