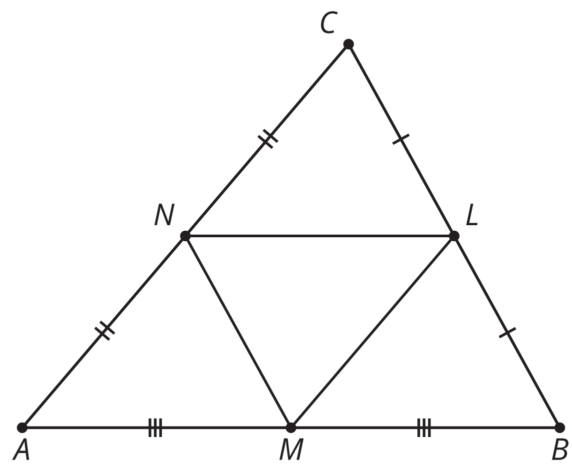
Splitting Triangle Sides with Dilation (Part 1)

Let's draw segments connecting midpoints of the sides of triangles.

5.1 Notice and Wonder: Midpoints

Here's a triangle ABC with midpoints L, M, and N. What do you notice? What do you wonder?

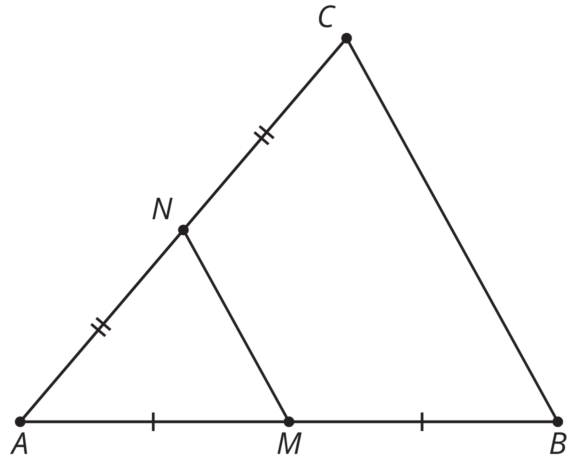
$\overline{BL} \cong \overline{CL}, \overline{AN} \cong \overline{CN}, \overline{AM} \cong \overline{BM}$



5.2 Dilation or Violation?

Here's a triangle ABC. Points M and N are the midpoints of 2 sides.

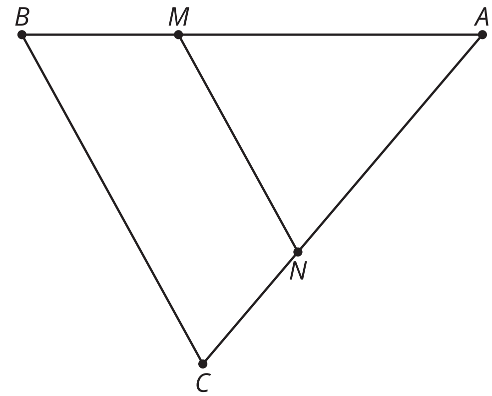
$\overline{AM} \cong \overline{BM}, \overline{AN} \cong \overline{CN}$



1. Convince yourself that triangle ABC is a dilation of triangle AMN. What is the center of the dilation? What is the scale factor?
2. Convince your partner that triangle ABC is a dilation of triangle AMN, with the center and scale factor that you found.
3. With your partner, check the definition of dilation on your reference chart, and make sure both of you could convince a skeptic that ABC definitely fits the definition of dilation.
4. Convince your partner that segment BC is twice as long as segment MN.
5. Prove that $BC = 2 \cdot MN$. Convince a skeptic.

5.3 A Little Bit Farther Now

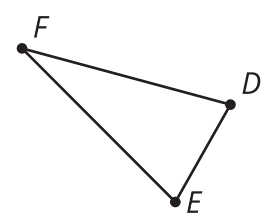
Here's a triangle, ABC. M is $\frac{2}{3}$ of the way from A to B. N is $\frac{2}{3}$ of the way from A to C.



What can you say about segment MN, compared to segment BC? Provide a reason for each of your conjectures.

Are you ready for more?

1. What do you think a negative scale factor could mean? Think about what happens to the point E after dilating with scale factors 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and so on getting closer to 0. Then think about what might happen with a zero or negative scale factor.
2. Dilate triangle DEF using a scale factor of -1 and center F.
3. How does DF compare to $D'F'$?
4. Are E, F, and E' collinear? Explain or show your reasoning.

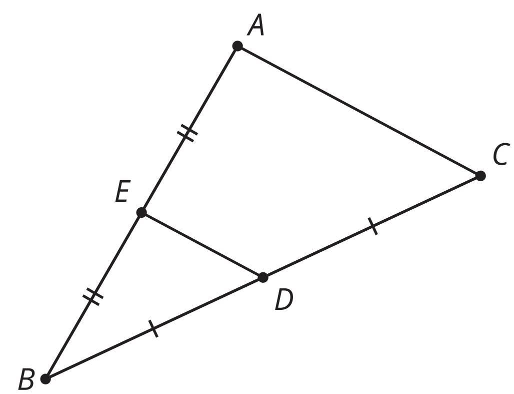


Lesson 5 Summary

Let's examine a segment whose endpoints are the midpoints of 2 sides of the triangle. If D is the midpoint of segment BC and E is the midpoint of segment BA, then what can we say about ED and triangle ABC?

Segment ED is parallel to the third side of the triangle and half the length of the third side of the triangle. For example, if $AC = 10$, then $ED = 5$. This happens because the entire triangle EBD is a dilation of triangle ABC, with a scale factor of $\frac{1}{2}$.

$\overline{BD} \cong \overline{DC}, \overline{BE} \cong \overline{EA}$



In triangle ABC, segment FG divides segments AB and CB proportionally. In other words, $\frac{BG}{GA} = \frac{BF}{FC}$. Again, there is a dilation that takes triangle ABC to triangle GBF, so GF is parallel to AC, and we can calculate its length using the same scale factor.

$\overline{FG} \parallel \overline{AC}$