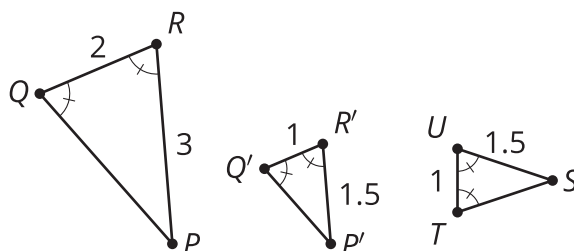# Lesson 9: Conditions for Triangle Similarity

- Let's prove some triangles similar.
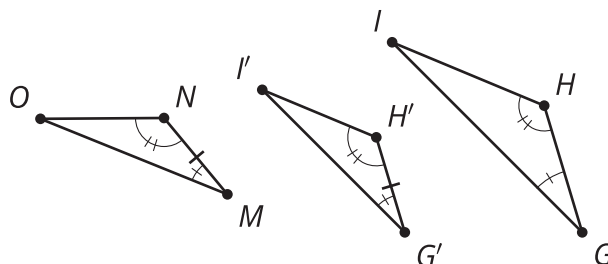
9.1: Math Talk: Angle-Side-Angle As A Helpful Tool

How could you justify each statement?

Triangle $P'Q'R'$ is congruent to triangle STU.

Triangle PQR is similar to triangle STU.

Triangle $G'H'I'$ is congruent to triangle MNO.

Triangle GHI is similar to triangle MNO.

9.2: How Many Pieces?

For each problem, draw 2 triangles that have the listed properties. Try to make them as different as possible.

1. One angle is 45 degrees.

2. One angle is 45 degrees and another angle is 30 degrees.

3. One angle is 45 degrees and another angle is 30 degrees. The lengths of a pair of corresponding sides are 2 cm and 6 cm.

4. Compare your triangles with your neighbors' triangles. Which ones seem to be similar no matter what?

5. Prove your conjecture.

9.3: Any Two Angles?

Here are 2 triangles. One triangle has a 60 degree angle and a 40 degree angle. The other triangle has a 40 degree angle and an 80 degree angle.
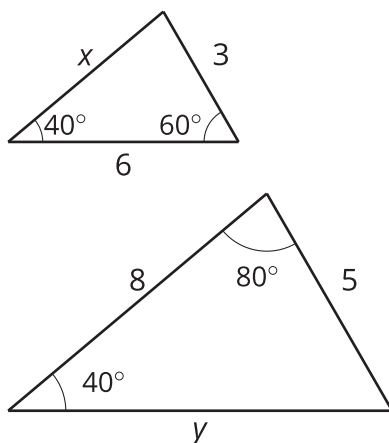
1. Explain how you know the triangles are similar.

2. How long are the sides labeled x and y?

Are you ready for more?

Under what conditions is there an Angle-Angle Quadrilateral Similarity Theorem? What about an Angle-Angle-Angle Quadrilateral Similarity Theorem? Explain or show your reasoning.

Lesson 9 Summary

When 2 angles of one triangle are congruent to 2 angles of a second triangle, the 2 triangles are similar. We call this the Angle-Angle Triangle Similarity Theorem.

In the diagram, angle A is congruent to angle D, and angle B is congruent to angle E. If a sequence of rigid motions and dilations moves the first figure so that it fits exactly over the second, then we have shown that the Angle-Angle Triangle Similarity Theorem is true.

$\angle A \cong \angle D, \angle B \cong \angle E$
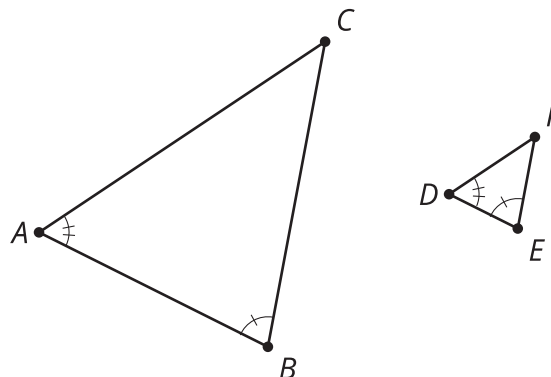
Dilate triangle ABC by the ratio $\frac{DE}{AB}$, so that $A'B'$ is congruent to DE. Now triangle $A'B'C'$ is congruent to triangle DEF by the Angle-Side-Angle Triangle Congruence Theorem, which means there is a sequence of rotations, reflections, and translations that takes $A'B'C'$ onto DEF.
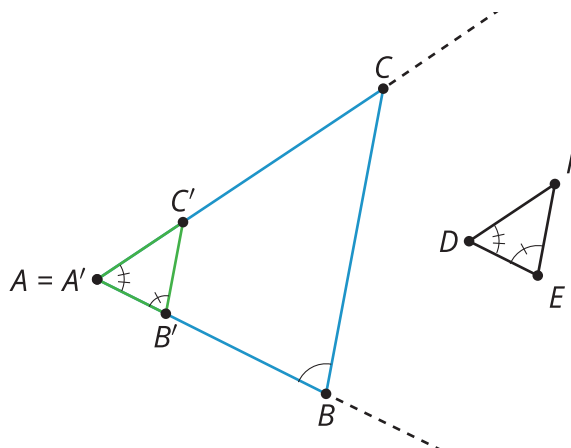
Therefore, a dilation followed by a sequence of rotations, reflections, and translations will take triangle ABC onto triangle DEF, which is the definition of similarity. We have shown that a dilation and a sequence of rigid motions takes triangle ABC to triangle DEF, so the triangles are similar.