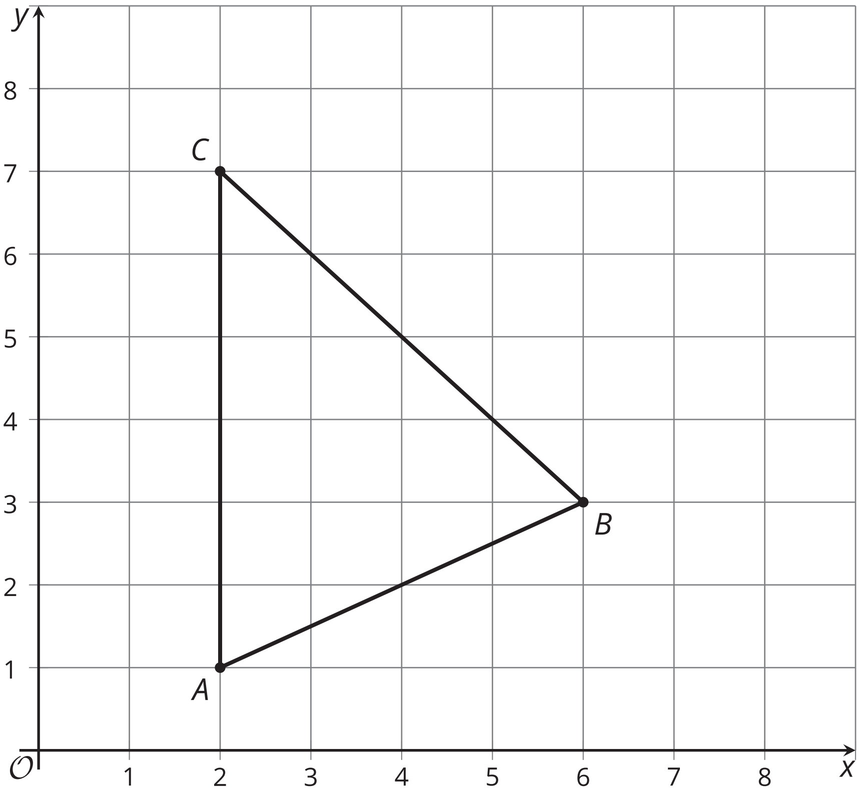
Illustrative Mathematics

Lesson 16: Weighted Averages in a Triangle

- Let’s partition special line segments in triangles.

16.1: Triangle Midpoints

Triangle ABC is graphed.



Find the midpoint of each side of this triangle.

16.2: Triangle Medians

Your teacher will tell you how to draw and label the **medians** of the triangle in the warm-up.

1. After the medians are drawn and labeled, measure all 6 segments inside the triangle using centimeters. What is the ratio of the 2 parts of each median?

2. Find the coordinates of the point that partitions segment AN in a $2:1$ ratio.

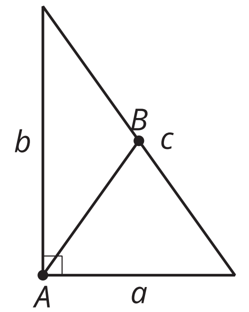
3. Find the coordinates of the point that partitions segment BL in a $2:1$ ratio.

4. Find the coordinates of the point that partitions segment CM in a $2:1$ ratio.

Are you ready for more?

In the image, AB is a median.

Find the length of AB in terms of a, b, and c.



16.3: Any Triangle's Medians

The goal is to prove that the medians of any triangle intersect at a point. Suppose the vertices of a triangle are $(0, 0)$, $(w, 0)$, and (a, b).

1. Each student in the group should choose 1 side of the triangle. If your group has 4 people, 2 can work together. Write an expression for the midpoint of the side you chose.

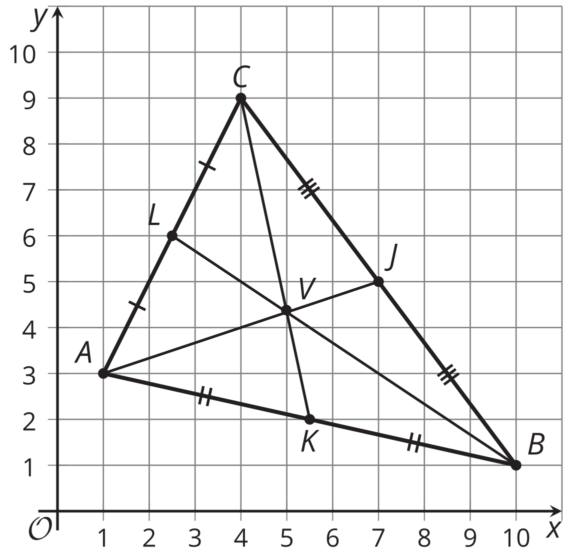
2. Each student in the group should choose a median. Write an expression for the point that partitions each median in a $2:1$ ratio from the vertex to the midpoint of the opposite side.

3. Compare the coordinates of the point you found to those of your groupmates. What do you notice?

4. Explain how these steps prove that the 3 medians of any triangle intersect at a single point.

Lesson 16 Summary

Here is a triangle with its **medians** drawn in. A median is a line segment drawn from a vertex of a triangle to the midpoint of the opposite side. Triangles have 3 medians, with 1 for each vertex.



Notice that the medians intersect at 1 point. This point is always $\frac{2}{3}$ of the distance from a vertex to the opposite midpoint. Another way to say this is that the point of intersection, V, partitions segments AJ, BL, and CK so that the ratios $AV : VJ$, $BV : VL$, and $CV : VK$ are all $2 : 1$.

We can prove this by working with a general triangle that can represent any triangle. Since any triangle can be transformed so that 1 vertex is on the origin and 1 side lies on the x-axis, we can say that our general triangle has vertices $(0, 0)$, $(w, 0)$, and (a, b). Through careful calculation, we can show that all 3 medians go through the point $\left(\frac{a+w}{3}, \frac{b}{3}\right)$. Therefore, the medians intersect at this point, which partitions each median in a $2 : 1$ ratio from the vertex to the opposite side's midpoint.