Unit 3, Lesson 2

Graphs of Rational Functions (Part 1)

Let's explore graphs and equations of rational functions.

2.1 Biking 10 Miles (Part 1)

Kiran's aunt plans to bike 10 miles.

1. How long will it take if she bikes at an average rate of 8 miles per hour?
2. How long will it take if she bikes at an average rate of r miles per hour?
3. Kiran wants to join his aunt, but he only has 45 minutes. What will their average rate need to be for him to finish on time?
4. What will their average rate need to be if they have t hours?

2.2 Biking 10 Miles (Part 2)

Kiran plans to bike 10 miles.

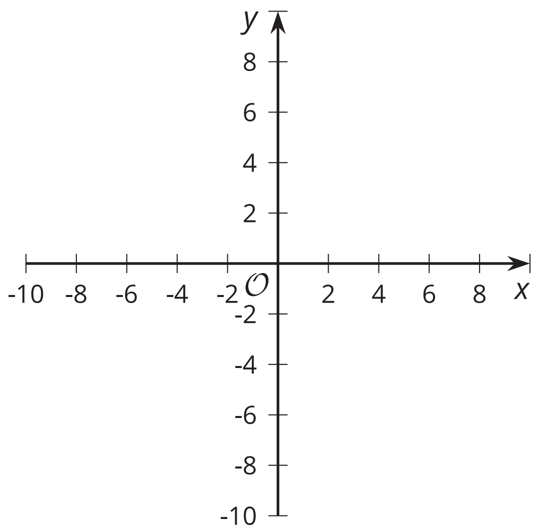
1. Write an equation that gives his time t, in hours, as a function of his rate r, in miles per hour.
2. Graph $y = t(r)$.
3. What is the meaning of $t(8)$? Does this value make sense? Explain your reasoning.
4. What is the meaning of $t(0)$? Does this value make sense? Explain your reasoning.
5. As r gets closer and closer to 0, what does the behavior of the function tell you about the situation? Be prepared to explain your reasoning.

2.3 Card Sort: Graphs of Rational Functions

Your teacher will give you a set of cards. Take turns with your partner to match an equation for a **rational function** with a graph that represents the same function.

Three of the cards have missing information. Complete the missing equations here:

- $v(x) = \underline{\qquad\qquad}$
- $q(x) = -\frac{1}{\underline{\qquad\qquad}}$
- Graph E:



Are you ready for more?

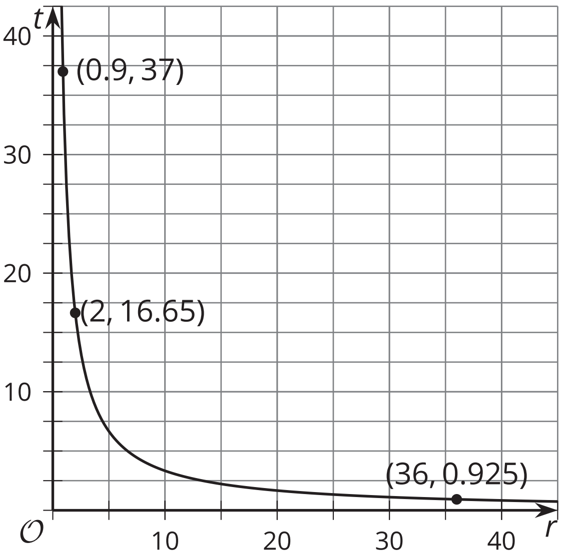
Priya and Han are bicycling. Han is going at a rate of 10 mph and begins 2 miles ahead of Priya. If Priya bikes at a rate of r mph, when will Priya pass Han? Explain or show your reasoning.

Lesson 2 Summary

The distance d that an object moving at constant speed travels is based on the length of time t the object travels and the speed r of the object. Often, this relationship is written as $d = r \cdot t$. We could also write the relationship as $r = \frac{d}{t}$ or $t = \frac{d}{r}$. Depending on what we want to know, one form of this relationship may be more useful than another.

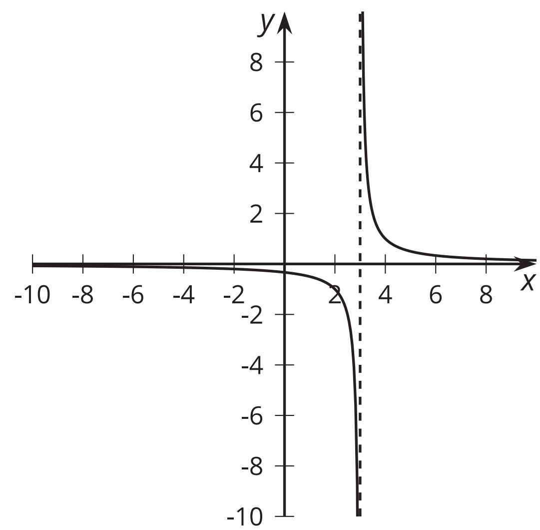
For example, the distance of a route crossing the English Channel from Dover in England to Calais in France is 33.3 kilometers. The time in hours it takes for a boat to make this crossing can be modeled by the function $T(r) = \frac{33.3}{r}$, where r is measured in kilometers per hour.

For very small values of r, the journey takes a long time since the boat is going very slow. For larger values of r (and a fast boat!), the trip is shorter. The graph of the function shows how the travel time decreases as the speed of the boat increases. When graphing, we also consider only positive values for r and t since both speed and time are positive in this situation.



Let's consider the graph from the other direction—that is, think about the output of the function from right to left. As the input approaches $r = 0$, the output increases rapidly. This makes sense because the slower the boat goes, the longer the trip is going to take. But what about at $r = 0$?

It turns out that even if we extended the graphing window vertically, we would never find an output for $r = 0$. A boat with a speed of 0 kilometers per hour will not cross the English Channel, so there is no output for the function at this input. This is an example of a feature of rational functions: a vertical asymptote.



A **vertical asymptote** is a value at which the function is undefined and the outputs of the function as it approaches the value of the asymptote get larger and larger in either the negative or positive direction. Vertical asymptotes can give some rational functions a disconnected look. For example, here is a graph of $f(x) = \frac{1}{x-3}$.

The dashed line at $x = 3$ is a representation of a vertical asymptote. As x gets closer and closer to 3, think about what happens to the value of the expression for $f(x)$. If we divide 1 by a very small negative number, we get a very big negative number, which is what happens on the left side of the vertical asymptote. If we divide 1 by a very small positive number, we get a very big positive number, which is what happens on the right side of the vertical asymptote. It is important to note that the drawn-in asymptote is not actually part of the graph of the function. Instead, it is a helpful reminder that the function has no value at $x = 3$ and very large absolute values at inputs very close to $x = 3$.

