Unit 8, Lesson 2

When and Why Do We Write Quadratic Equations?

Let's try to solve some quadratic equations.

2.1 How Many Tickets?

The expression $12t + 2.50$ represents the cost to purchase tickets for a play, where t is the number of tickets. Be prepared to explain your response to each question.

1. A family paid $62.50 for tickets. How many tickets were bought?

2. A teacher paid $278.50 for tickets for her students. How many tickets were bought?

2.2 The Flying Potato Again

Here is a function modeling the height of a potato, in feet, t seconds after being fired from a device:

$$f(t) = -16t^2 + 80t + 64$$

1. What equation would we solve to find the time at which the potato hits the ground?

2. Use any method *except graphing* to find a solution to this equation.

2.3 Revenue from Ticket Sales

The expressions $p(200 - 5p)$ and $-5p^2 + 200p$ describe the revenue a school would earn from selling raffle tickets at p dollars each.

1. For each situation, write a **quadratic equation** using these quadratic expressions. Then, find the price, p, of each ticket that would produce the situation. Explain your reasoning.
   a. The school collects $0 in revenue from raffle sales.

   b. The school collects $500 in revenue from raffle sales.

Are you ready for more?

Can you find the following prices without graphing?

1. If the school charges $10, it will collect $1,500 in revenue. Find another price that would generate $1,500 in revenue.

2. If the school charges $28, it will collect $1,680 in revenue. Find another price that would generate $1,680 in revenue.

3. Find the price that would produce the maximum possible revenue. Explain your reasoning.

Lesson 2 Summary

The height of a potato that is launched from a mechanical device can be modeled by a function, g, with x representing time in seconds. Here are two expressions that are equivalent and both define function g.

$$-16x^2 + 80x + 96 \qquad -16(x - 6)(x + 1)$$

Notice that one expression is in *standard form* and the other is in *factored form*.

Suppose we wish to know, without graphing the function, the time when the potato will hit the ground. We know that the value of the function at that time is 0, so we can write:

$$-16x^2 + 80x + 96 = 0 \qquad -16(x - 6)(x + 1) = 0$$

Let's try solving $-16x^2 + 80x + 96 = 0$, using some familiar moves. For example:

- Subtract 96 from each side: $-16x^2 + 80x = -96$
- Apply the distributive property to rewrite the expression on the left: $-16(x^2 - 5x) = -96$
- Divide both sides by -16: $x^2 - 5x = 6$
- Apply the distributive property to rewrite the expression on the left: $x(x - 5) = 6$

These steps don't seem to get us any closer to a solution. We need some new moves!

What if we use the other equation? Can we find the solutions to $-16(x - 6)(x + 1) = 0$?

Earlier, we learned that the *zeros* of a quadratic function can be identified when the expression defining the function is in factored form. The solutions to $-16(x - 6)(x + 1) = 0$ are the zeros to function g, so this form may be more helpful! We can reason that:

- If x is 6, then the value of $x - 6$ is 0, so the entire expression has a value of 0.
- If x is -1, then the value of $x + 1$ is 0, so the entire expression also has a value of 0.

This tells us that 6 and -1 are solutions to the equation, and that the potato hits the ground after 6 seconds. (A negative value of time is not meaningful, so we can disregard the -1.)

Both equations we see here are quadratic equations. In general, a **quadratic equation** is an equation that can be expressed as $ax^2 + bx + c = 0$, where a, b, and c are constants and $a \neq 0$.

In upcoming lessons, we will learn how to rewrite quadratic equations into forms that make the solutions easy to see.