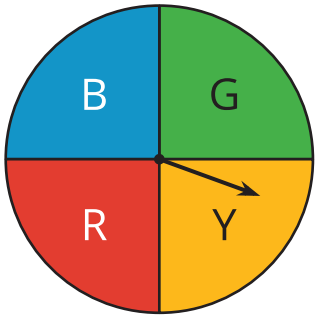
Estimating Probabilities Using Simulation

Let's simulate real-world situations.

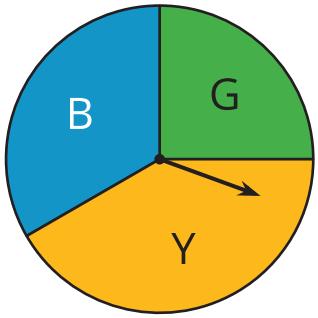
6.1 Which Three Go Together: Spinners

Which three go together? Why do they go together?

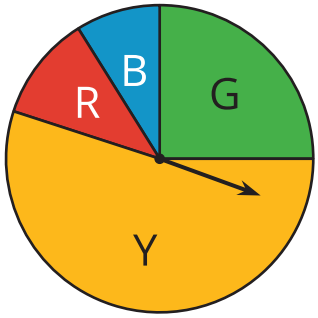
A



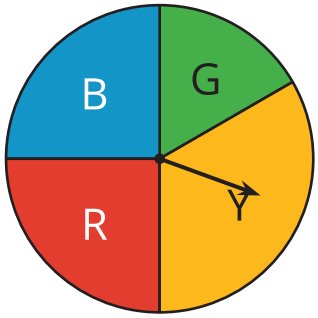
B



C



D

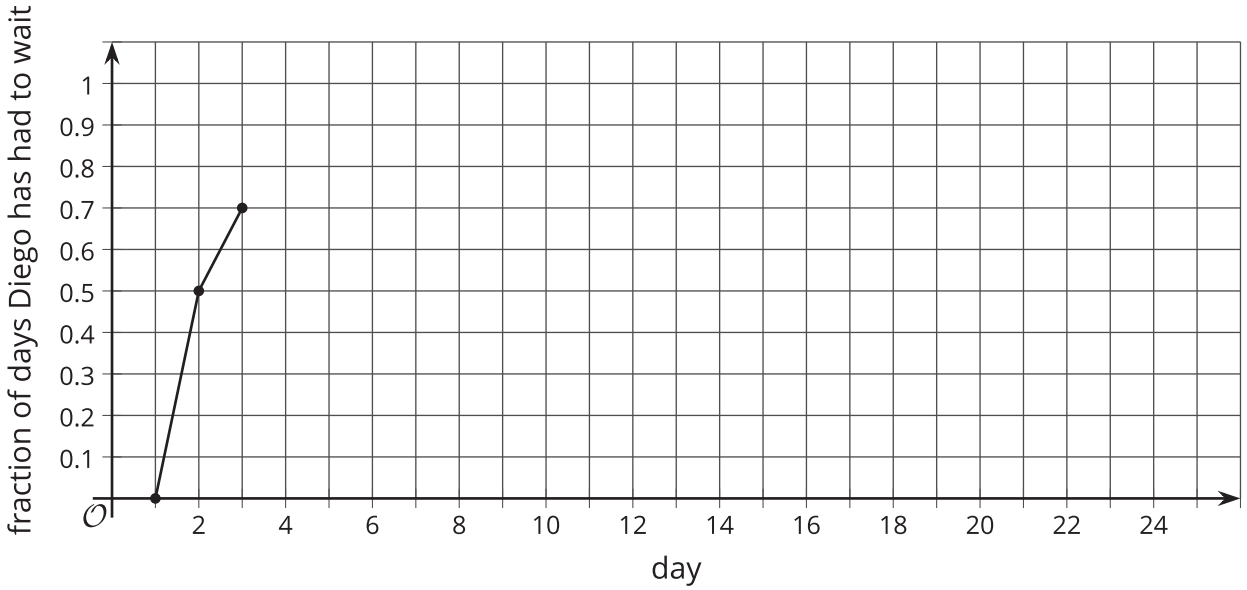


CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

6.2 Diego's Walk

Your teacher will give your group the supplies for one of the three different **simulations**. Follow these instructions to simulate 15 days of Diego's walk. The first 3 days have been done for you.

- Simulate one day:
  - If your group gets a bag of papers, reach into the bag, and select one paper without looking inside.
  - If your group gets a spinner, spin the spinner, and see where it stops.
  - If your group gets two number cubes, roll both cubes, and add the numbers that land face up. A sum of 2–8 means Diego has to wait.
- Record in the table whether or not Diego has to wait more than 1 minute.
- Calculate the total number of days and the cumulative fraction of days that Diego has had to wait so far.
- On the graph, plot the number of days and the fraction that Diego has had to wait. Connect each point by a line.
- If your group has the bag of papers, put the paper back into the bag, and shake the bag to mix up the papers.
- Pass the supplies to the next person in the group and they will repeat the steps to simulate the next day.



CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

day	Does Diego have to wait more than 1 minute?	total number of days Diego has had to wait	fraction of days Diego has had to wait
1	no	0	$\frac{0}{1} = 0.00$
2	yes	1	$\frac{1}{2} = 0.50$
3	yes	2	$\frac{2}{3} \approx 0.67$
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

1. Based on the data you have collected, do you think the fraction of days Diego has had to wait after the 16th day will be closer to 0.9 or 0.7? Explain or show your reasoning.

2. Continue the simulation for 10 more days. Record your results in this table and on the graph from earlier.

day	Does Diego have to wait more than 1 minute?	total number of days Diego has had to wait	fraction of days Diego has had to wait
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

3. What do you notice about the graph?

4. Based on the graph, estimate the probability that Diego will have to wait more than 1 minute to cross the crosswalk.

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

Are you ready for more?

Let's look at why the values tend to not change much after doing the simulation many times.

1. After doing the simulation 4 times, a group finds that Diego had to wait 3 times. What is an estimate for the probability Diego has to wait based on these results?

    a. If this group does the simulation 1 more time, what are the two possible outcomes for the fifth simulation?

    b. For each possibility, estimate the probability Diego has had to wait.

    c. What are the differences between the two possible probability estimates after 5 simulations and the estimate after 4 simulations?

2. After doing the simulation 20 times, this group finds that Diego has had to wait 15 times. What is an estimate for the probability Diego has to wait based on these results?

    a. If this group does the simulation 1 more time, what are the two possible outcomes for the twenty-first simulation?

    b. For each possibility, estimate the probability Diego has to wait.

    c. What are the differences between the possible estimates after 21 simulations and the estimate after 20 simulations?

3. Use these results to explain why a single result after many simulations does not affect the estimate as much as a single result after only a few simulations.

6.3 Designing Experiments

For each situation, describe a chance experiment that would fairly represent it.

1. Six people are going out to lunch together. One of them will be selected at random to choose which restaurant to go to. Who gets to choose?

2. After a robot stands up, it is equally likely to step forward with its left foot or its right foot. Which foot will it use for its first step?

3. In a computer game, there are 3 tunnels. Each time the level loads, the computer randomly selects 1 of the tunnels to lead to the castle. Which tunnel is it?

4. Your school is taking 4 buses of students on a field trip. Will you be assigned to the same bus that your math teacher is riding on?

Lesson 6 Summary

Sometimes it is easier to estimate a probability by doing a **simulation**. A simulation is an experiment that approximates a situation in the real world. Simulations are useful when it is hard or time-consuming to gather enough information to estimate the probability of some event.

For example, imagine Andre has to transfer from one bus to another on the way to his music lesson. Most of the time he makes the transfer just fine, but sometimes the first bus is late to the transfer point, and he misses the second bus. We could set up a simulation with slips of paper in a bag. For each bus ride over the last month, we write either "miss" or "made it" on a slip of paper depending on whether he missed the transfer or not. We select slips at random from the bag. After many trials, we calculate the fraction of the times that he misses the second bus to estimate the probability that he will miss the bus on a given day.