Unit 5, Lesson 17

Applying the Quadratic Formula (Part 1)

Let’s use the quadratic formula to solve some problems.

17.1 No Solutions for You!

Here is an example of someone solving a quadratic equation that has no solutions:

$$(x+3)^2+9=0$$
$$(x+3)^2=-9$$
$$x+3=\pm\sqrt{-9}$$

1. Study the example. At what point did you realize the equation had no solutions?

2. Explain how you know the equation $49+x^2=0$ has no solutions.

17.2 The Potato and the Pumpkin

Answer each question without graphing. Explain or show your reasoning.

1. The equation $h(t) = -16t^2 + 80t + 64$ represents the height, in feet, of a potato t seconds after it has been launched.

   a. Write an equation that can be solved to find when the potato hits the ground. Then solve the equation.

   b. Write an equation that can be solved to find when the potato is 40 feet off the ground. Then solve the equation.

2. The equation $g(t) = 2 + 23.7t - 4.9t^2$ models the height, in meters, of a pumpkin t seconds after it has been launched from a catapult.

   a. Is the pumpkin still in the air 8 seconds later? Explain or show how you know.

   b. At what value of t does the pumpkin hit the ground? Show your reasoning.

17.3 Back to the Framer

1. Solve this equation without graphing. $(7 + 2x)(4 + 2x) = 38$.

   Pause for a discussion about the equation.

2. Suppose you have another picture that is 10 inches by 5 inches, and are now using a fancy paper that is 8.5 inches by 4 inches to frame the picture. Again, the frame is to be uniform in thickness all the way around. No fancy framing paper is to be wasted!

   Find out how thick the frame should be.

Are you ready for more?

Suppose that your border paper is 6 inches by 8 inches. You want to use all the paper to make a half-inch border around a rectangular picture.

1. Find two possible pairs of length and width of a rectangular picture that could be framed with a half-inch border and no leftover materials.

2. What must be true about the length and width of any rectangular picture that can be framed this way? Explain how you know.

Lesson 17 Summary

Quadratic equations that represent situations cannot always be neatly put into factored form or easily solved by finding square roots. Completing the square is a workable strategy, but for some equations, it may involve many cumbersome steps. Graphing is also a handy way to solve the equations, but it doesn't always give us precise solutions.

With the quadratic formula, we can solve these equations more readily and precisely.

Here's an example: Function h models the height of an object, in meters, t seconds after it is launched into the air. It is is defined by $h(t) = -5t^2 + 25t$.

To know how much time it would take the object to reach 15 meters, we could solve the equation $15 = -5t^2 + 25t$. How should we do it?

- Rewriting it in standard form gives $-5t^2 + 25t - 15 = 0$. The expression on the left side of the equation cannot be written in factored form, however.
- Completing the square isn't convenient because the coefficient of the squared term is not a perfect square and the coefficient the linear term is an odd number.
- Let's use the quadratic formula, using $a = -5, b = 25, \text{and } c = -15$!

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-25 \pm \sqrt{25^2 - 4(-5)(-15)}}{2(-5)}$$

$$t = \frac{-25 \pm \sqrt{325}}{-10}$$

The expression $\frac{-25 \pm \sqrt{325}}{-10}$ represents the two exact solutions of the equation.

We can also get approximate solutions by using a calculator, or by reasoning that $\sqrt{325} \approx 18$.

The solutions tell us that there are two times after the launch when the object is at a height of 15 meters: at about 0.7 second (as the object is going up) and 4.3 seconds (as it comes back down).

