

## Proving the Triangle Congruence Theorems

### Sentence Frames for Proofs

#### Transformations:

- Translate \_\_\_\_\_ from \_\_\_\_\_ to \_\_\_\_\_.
- Rotate \_\_\_\_\_ using \_\_\_\_\_ as the center by angle \_\_\_\_\_.
- Rotate \_\_\_\_\_ using \_\_\_\_\_ as the center so that \_\_\_\_\_ coincides with \_\_\_\_\_.
- Reflect \_\_\_\_\_ across \_\_\_\_\_.
- Reflect \_\_\_\_\_ across the perpendicular bisector of \_\_\_\_\_.
- Segments \_\_\_\_\_ and \_\_\_\_\_ are the same length so they are congruent. Therefore, there is a rigid motion that takes \_\_\_\_\_ to \_\_\_\_\_. Apply that rigid motion to \_\_\_\_\_.

#### Justifications:

- We know the image of \_\_\_\_\_ is congruent to \_\_\_\_\_ because rigid motions preserve measure.
- Points \_\_\_\_\_ and \_\_\_\_\_ coincide after translating because we defined our translation that way!
- Since points \_\_\_\_\_ and \_\_\_\_\_ are the same distance along the same ray from \_\_\_\_\_ they have to be in the same place.
- Rays \_\_\_\_\_ and \_\_\_\_\_ coincide after rotating because we defined our rotation that way!
- The image of \_\_\_\_\_ must be on ray \_\_\_\_\_ since both \_\_\_\_\_ and \_\_\_\_\_ are on the same side of \_\_\_\_\_ and make the same angle with it at \_\_\_\_\_.
- Points \_\_\_\_\_ and \_\_\_\_\_ coincide because they are both at the intersection of the same lines, and 2 distinct lines can only intersect in 1 place.
- \_\_\_\_\_ is the perpendicular bisector of the segment connecting \_\_\_\_\_ and \_\_\_\_\_, because the perpendicular bisector is determined by 2 points that are both equidistant from the endpoints of a segment.

#### Conclusion statement:

- We have shown that a rigid motion takes \_\_\_\_\_ to \_\_\_\_\_, \_\_\_\_\_ to \_\_\_\_\_, and \_\_\_\_\_ to \_\_\_\_\_, therefore triangle \_\_\_\_\_ is congruent to triangle \_\_\_\_\_.

## Using the Triangle Congruence Theorems

## More Proof Supports

Many proofs in Euclidean geometry that don't use transformations use congruent triangles: if you can find two triangles that you are SURE are congruent, you can prove that any corresponding parts of your triangles are congruent.

1. Can you find any triangles that are probably congruent? Suggestion: outline them in different colors or re-draw them separately on your paper.
2. If you can't find any triangles yet, is there a helpful auxiliary line you can draw?
  - a. A line of symmetry?
  - b. A segment connecting two points, such as the diagonal of a quadrilateral?
3. Label all of the things you know are congruent. This will help you decide how to prove two triangles are congruent.
  - a. Do you know all three pairs of corresponding sides are congruent? Use SSS Congruence!
  - b. Do you know two pairs of corresponding angles are congruent? Look to see if you can show the sides between the corresponding angles are congruent to use ASA Congruence!
  - c. Do you know two pairs of corresponding sides are congruent? Look to see if you can show the angles between the corresponding sides are congruent to use SAS!
4. Seems like there's not enough information? Here are some things to check:
  - a. Do the triangles share a side or an angle? Sides and angles are congruent to themselves!
  - b. Are any of the sides radii of the same circle? All of the radii in the same circle are congruent.
  - c. Are there parallel lines? Look for angles that must be congruent when formed by parallel lines, such as alternate interior angles.
  - d. Are there vertical angles?
  - e. Is there a quadrilateral with special properties?

You can use this template if you want:

Goal: Prove \_\_\_\_\_ is congruent to \_\_\_\_\_

I'm going to do this by proving Triangle \_\_\_\_\_ is congruent to triangle \_\_\_\_\_ by \_\_\_\_\_ Congruence Theorem.

Statement 1:

Reason 1:

Statement 2:

Reason 2:

Statement 3:

Reason 3:

Therefore, Triangle \_\_\_\_\_ is congruent to triangle \_\_\_\_\_ by \_\_\_\_\_ Congruence Theorem.

Since \_\_\_\_\_ and \_\_\_\_\_ are corresponding parts of congruent triangles, \_\_\_\_\_ and \_\_\_\_\_ must be congruent.