The Distributive Property, Part 2

Let's use rectangles to understand the distributive property with variables.

9.1 Possible Areas

1. A rectangle has a length of 4 units and a width of m units. Write an expression for the area of this rectangle.

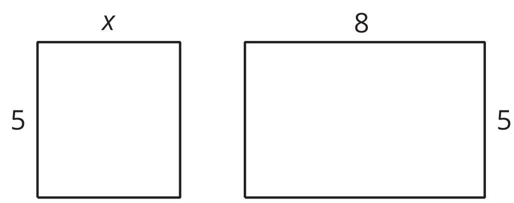
2. What is the area of the rectangle if m is:

   3 units? 2.2 units? $\frac{1}{5}$ unit?

3. Could the area of this rectangle be 11 square units? Explain your reasoning.

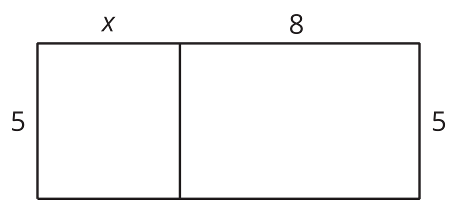
9.2 Partitioned Rectangles When Lengths Are Unknown

1. Here are two rectangles. The length and width of one rectangle are 8 and 5 units. The width of the other rectangle is 5 units, but its length is unknown so we labeled it x.



Write an expression for the sum of the areas of the two rectangles.

2. The two rectangles can be composed into one larger rectangle, as shown.



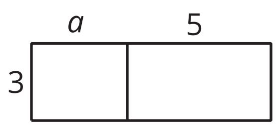
What are the length and width of the new, larger rectangle?

3. Write an expression for the total area of the new, larger rectangle as the product of its width and its length.

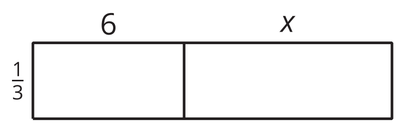
9.3 Areas of Partitioned Rectangles

For each rectangle, write an expression for the width, an expression for the length, and two expressions for the total area. Record them in the table. Check your expressions in each row with your group and discuss any disagreements.

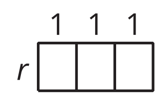
A



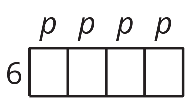
B



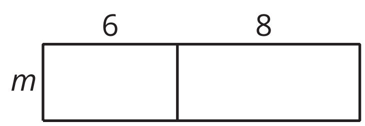
C



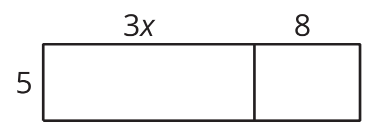
D



E



F

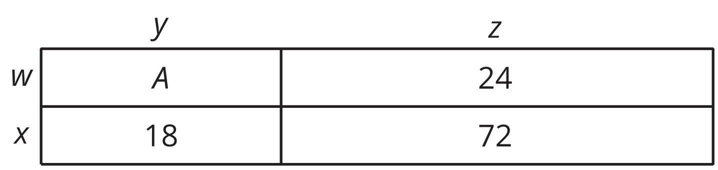


rectangle	width	length	area as a product of width times length	area as a sum of the areas of the smaller rectangles
A				
B				
C				
D				
E				
F				

Are you ready for more?

Here is a diagram showing a rectangle partitioned into four smaller rectangles.

- The variables w, x, y, and z represent lengths and widths of those smaller rectangles.
- The variable A and the numbers 24, 18, and 72 represent the areas.
- All values for variables are whole numbers.



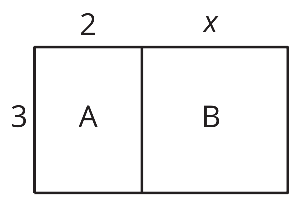
1. Find the values of w, x, y, z, and A. (The measurements of the lengths in the diagram do not match the values of the variables, so measuring them will not help.)

2. Can you find another set of lengths that will work? How many possibilities are there?

Lesson 9 Summary

The distributive property can also help us write equivalent expressions with variables. We can use a diagram to help us understand this idea.

Here is a rectangle composed of two smaller Rectangles A and B.



Based on the drawing, we can make several observations about the area of the large rectangle:

- One side length of the large rectangle is 3 and the other is $2 + x$, so its area is $3(2 + x)$.
- Since the large rectangle can be decomposed into two smaller rectangles, A and B, with no overlap, the area of the large rectangle is also the sum of the areas of rectangles A and B: $3 \cdot 2 + 3 \cdot x$ or $6 + 3x$.
- Since both expressions represent the area of the large rectangle, they are equivalent to each other. $3(2 + x)$ is equivalent to $6 + 3x$.

We can see that multiplying 3 by the sum $2 + x$ is equivalent to multiplying 3 by 2 and then 3 by x and adding the two products. This relationship is an example of the distributive property.

$$3(2 + x) = 3 \cdot 2 + 3 \cdot x$$

When working with expressions of all kinds, it helps to be able to talk about the parts. In an expression like $6 + 3x$, we call the 6 and $3x$ "terms."

A **term** is a part of an expression. A term can be a single number, a single variable, or a product of numbers and variables. Some examples of terms are 10, $8x$, ab, and $7yz$.