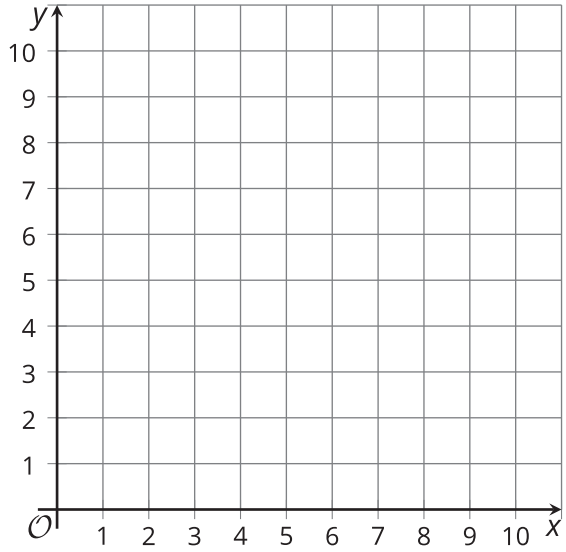
Unit 2, Lesson 12

# Weighted Averages

Let's split segments using averages and ratios.

## 12.1 Part Way: Points

For the questions in this activity, use the coordinate grid if it is helpful to you.




1. What is the midpoint of the segment connecting $(1, 2)$ and $(5, 2)$?

2. What is the midpoint of the segment connecting $(5, 2)$ and $(5, 10)$?

3. What is the midpoint of the segment connecting $(1, 2)$ and $(5, 10)$?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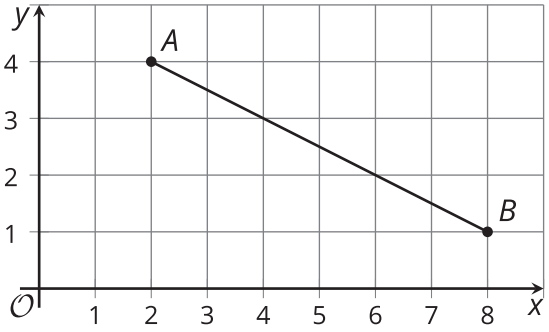
# 12.2 Part Way: Segment

Point $A$ has coordinates $(2, 4)$. Point $B$ has coordinates $(8, 1)$.




1. Find the point that partitions segment $AB$ in a $2 : 1$ ratio.

2. Calculate $C = \frac{1}{3}A + \frac{2}{3}B$.

3. What do you notice about your answers to the first 2 questions?

4. For 2 new points $K$ and $L$, write an expression for the point that partitions segment $KL$ in a $3 : 1$ ratio.

## Are you ready for more?

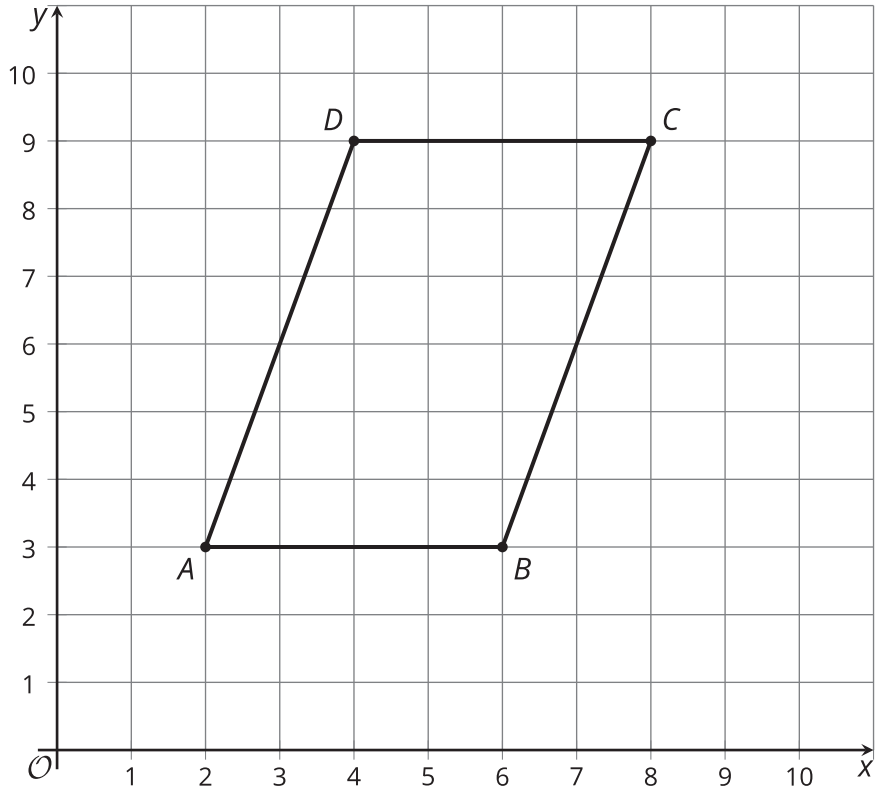
Consider the general quadrilateral $QRST$ with $Q(0, 0)$, $R(a, b)$, $S(c, d)$, and $T(e, f)$.

1. Find the midpoints of each side of this quadrilateral.

2. Show that if these midpoints are connected consecutively, the new quadrilateral formed is a parallelogram.

# 12.3 Part Way: Quadrilateral

Here is quadrilateral $ABCD$.




1. Find the point that partitions segment $AB$ in a $1:4$ ratio. Label it $B'$.

2. Find the point that partitions segment $AD$ in a $1:4$ ratio. Label it $D'$.

3. Find the point that partitions segment $AC$ in a $1:4$ ratio. Label it $C'$.

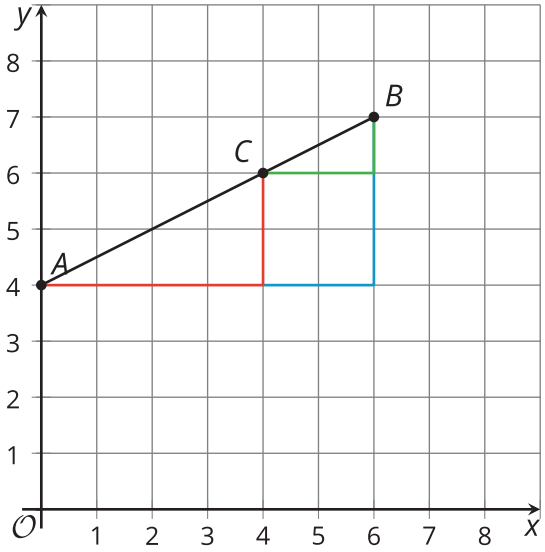
4. Is $AB'C'D'$ a dilation of $ABCD$? Justify your answer.

## Lesson 12 Summary

To find the midpoint of a line segment, we can average the coordinates of the endpoints. For example, to find the midpoint of the segment from $A(0, 4)$ to $B(6, 7)$, average the coordinates of $A$ and $B$: $\left(\frac{0+6}{2}, \frac{4+7}{2}\right) = (3, 5.5)$. Another way to write what we just did is $\frac{1}{2}(A + B)$ or $\frac{1}{2}A + \frac{1}{2}B$.

Now, let's find the point that is $\frac{2}{3}$ of the way from $A$ to $B$. In other words, we'll find point $C$ so that segments $AC$ and $CB$ are in a $2 : 1$ ratio.

In the horizontal direction, segment $AB$ stretches from $x = 0$ to $x = 6$. The distance from 0 to 6 is 6 units, so we calculate $\frac{2}{3}$ of 6 to get 4. Point $C$ will be 4 horizontal units away from $A$, which means an $x$-coordinate of 4.




In the vertical direction, segment $AB$ stretches from $y = 4$ to $y = 7$. The distance from 4 to 7 is 3 units, so we can calculate $\frac{2}{3}$ of 3 to get 2. Point $C$ must be 2 vertical units away from $A$, which means a $y$-coordinate of 6.

It is possible to do this all at once by saying $C = \frac{1}{3}A + \frac{2}{3}B$. This is called a weighted average. Instead of finding the point in the middle, we want to find a point closer to $B$ than to $A$. So we give point $B$ more weight—it has a coefficient of $\frac{2}{3}$ rather than $\frac{1}{2}$ as in the midpoint calculation. To calculate $C = \frac{1}{3}A + \frac{2}{3}B$, substitute and evaluate.

$$\frac{1}{3}A + \frac{2}{3}B$$

$$\frac{1}{3}(0, 4) + \frac{2}{3}(6, 7)$$

$$\left(0, \frac{4}{3}\right) + \left(4, \frac{14}{3}\right)$$

$$(4, 6)$$

Either way, we found that the coordinates of $C$ are $(4, 6)$.