

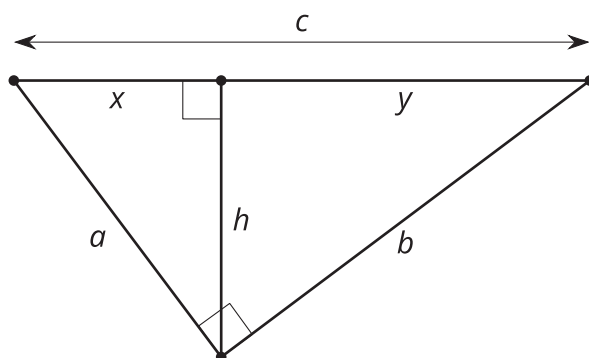


# Proving the Pythagorean Theorem

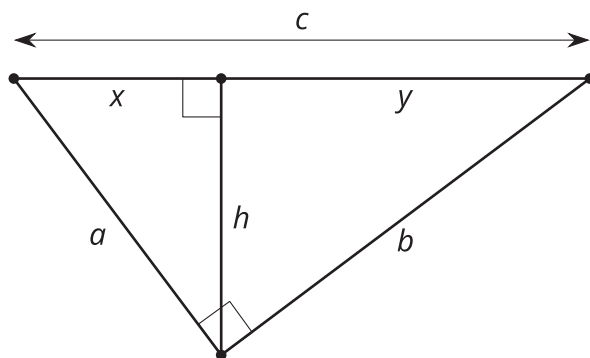
Let's prove the Pythagorean Theorem.

## 16.1 Notice and Wonder: Variable Version

What do you notice? What do you wonder?



## 16.2 Prove Pythagoras Right

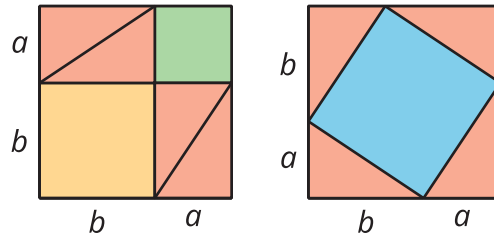


Elena is playing with the equivalent ratios she wrote using this diagram. She rewrites  $\frac{a}{x} = \frac{c}{a}$  as  $a^2 = xc$ . Diego notices and comments, "I got  $b^2 = yc$ . The  $a^2$  and  $b^2$  remind me of the Pythagorean Theorem." Elena says, "The Pythagorean Theorem says that  $a^2 + b^2 = c^2$ . I bet we could figure out how to show that."

1. How did Elena get from  $\frac{a}{x} = \frac{c}{a}$  to  $a^2 = xc$ ?
2. What equivalent ratios of side lengths did Diego use to get  $b^2 = yc$ ?
3. Prove  $a^2 + b^2 = c^2$  in a right triangle with legs length  $a$  and  $b$  and hypotenuse length  $c$ .

## 16.3

## An Alternate Approach



When Pythagoras proved his theorem, he used the two images shown here. Can you figure out how he used these diagrams to prove that  $a^2 + b^2 = c^2$  in a right triangle with a hypotenuse of length  $c$ ?

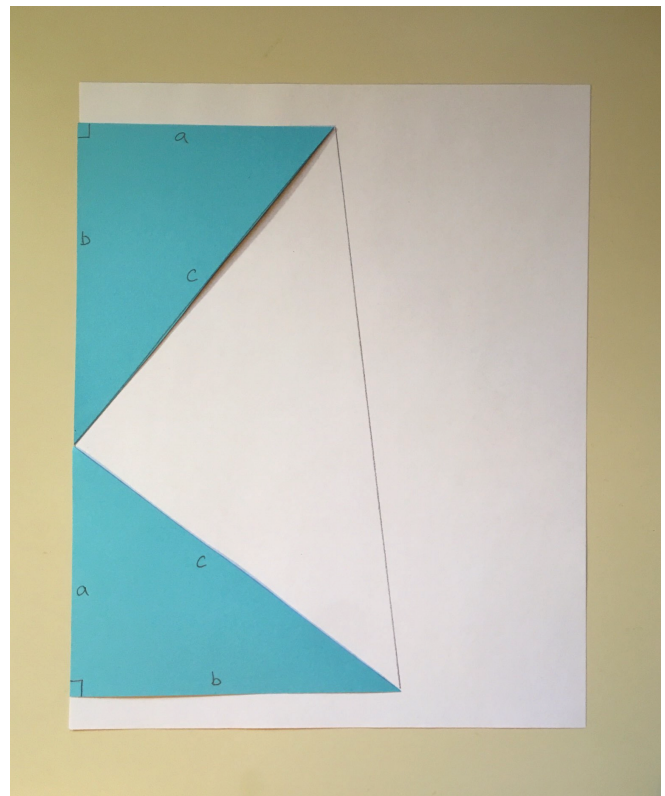


## Are you ready for more?

James Garfield, the 20th president, proved the Pythagorean Theorem in a different way.

- Cut out 2 congruent right triangles
- Label the long sides  $b$ , the short sides  $a$  and the hypotenuses  $c$ .
- Align the triangles on a piece of paper, with one long side and one short side in a line. Draw the line connecting the other acute angles.

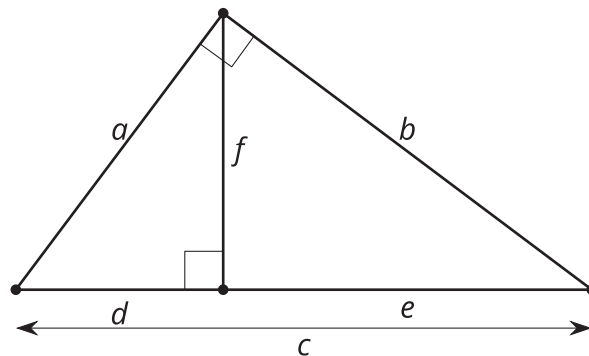
How does this diagram prove the Pythagorean Theorem?



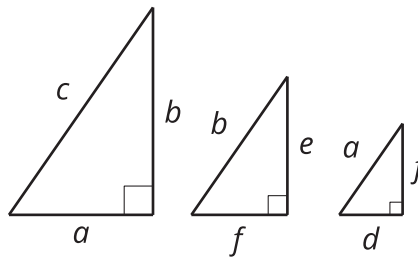
## Lesson 16 Summary

In any right triangle with legs  $a$  and  $b$  and hypotenuse  $c$ , we know that  $a^2 + b^2 = c^2$ . We call this the Pythagorean Theorem. But why does it work?

We can use an altitude drawn to the hypotenuse of a right triangle to prove the Pythagorean Theorem.



We can use the Angle-Angle Triangle Similarity Theorem to show that all 3 triangles are similar. Because the triangles are similar, the corresponding side lengths are in the same proportion.



Because the largest triangle is similar to the smaller triangle,  $\frac{c}{a} = \frac{a}{d}$ . Because the largest triangle is similar to the middle triangle,  $\frac{c}{b} = \frac{b}{e}$ . We can rewrite these equations as  $a^2 = cd$  and  $b^2 = ce$ .

We can add the 2 equations to get that  $a^2 + b^2 = cd + ce$ , or  $a^2 + b^2 = c(d + e)$ . From the original diagram we can see that  $d + e = c$ , so  $a^2 + b^2 = c(c)$ , or  $a^2 + b^2 = c^2$ .