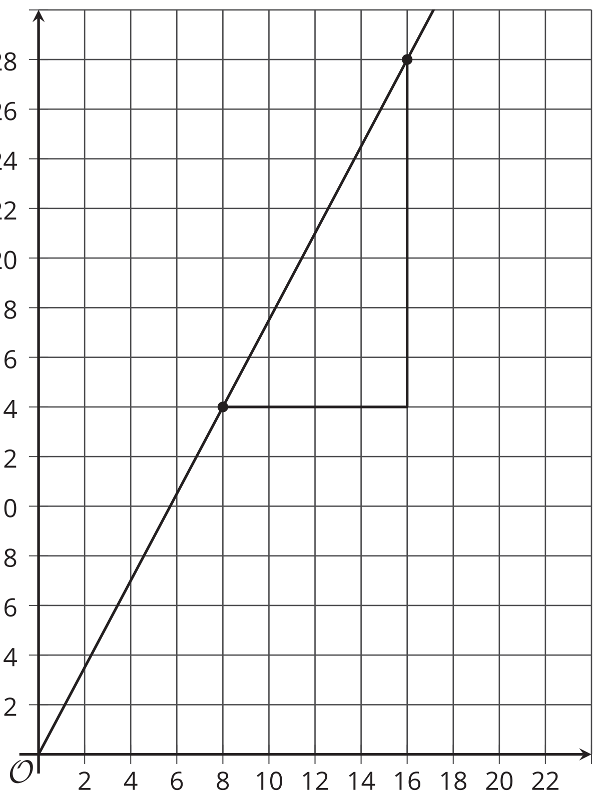
Unit 5, Lesson 2

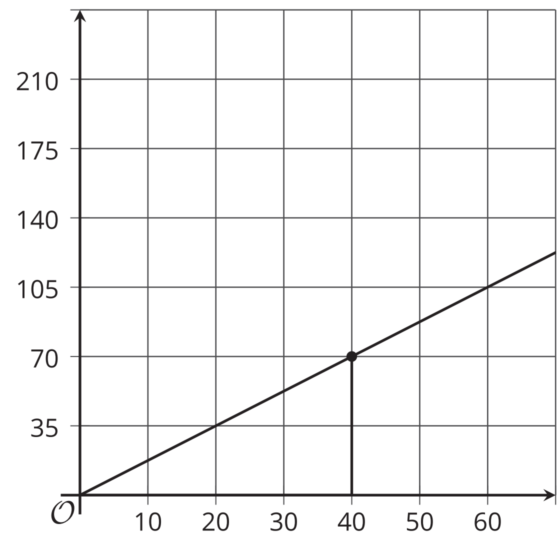
Representing Proportional Relationships

Let's graph proportional relationships.

2.1 Two Perspectives

Here are two graphs that could represent a variety of different situations.





Andre claims that the line in the graph on the left has a greater slope because it is steeper. Do you agree with Andre? Explain your reasoning.

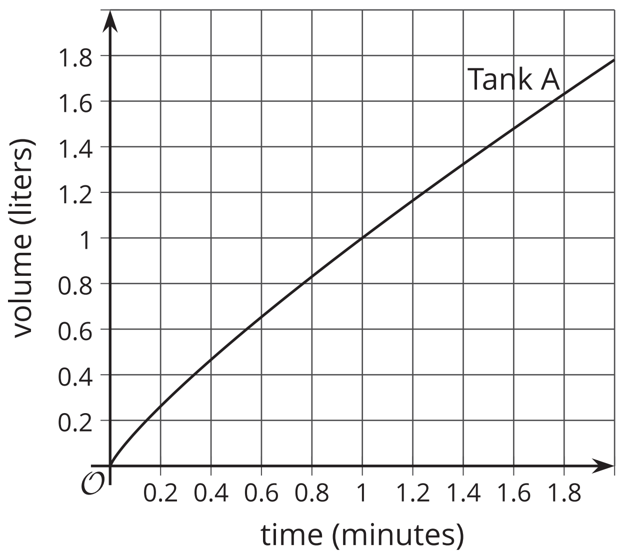
2.2 Card Sort: Proportional Relationships

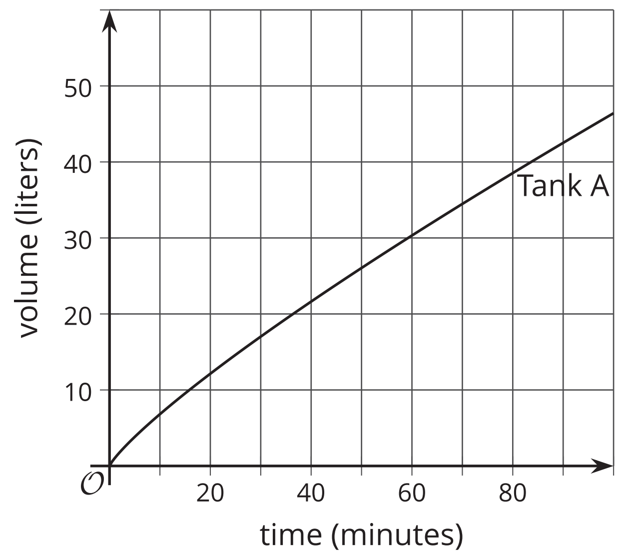
Your teacher will give you a set of cards. Each card contains a graph of a proportional relationship.

1. Sort the graphs into groups based on what proportional relationship they represent.
2. Write an equation for each *different* proportional relationship you find.

2.3 Different Scales

Two large water tanks are filling with water. Tank A is *not* filled at a constant rate, and the relationship between its volume of water and time is graphed on each set of axes. Tank B is filled at a constant rate of $\frac{1}{2}$ liters per minute. The relationship between its volume of water and time can be described by the equation $v = \frac{1}{2}t$, where t is the time in minutes, and v is the total volume in liters of water in the tank.





1. Sketch and label a graph of the relationship between the volume of water v and time t for Tank B on each of the coordinate planes.

2. Answer the following questions and say which graph you used to find your answer.

   a. After 30 seconds, which tank has the most water?

   b. At approximately what times do both tanks have the same amount of water?

   c. At approximately what times do both tanks contain 1 liter of water? 20 liters?

Are you ready for more?

A giant tortoise travels at 0.17 miles per hour and an arctic hare travels at 37 miles per hour.

1. Draw separate graphs that show the relationship between time elapsed, in hours, and distance traveled, in miles, for both the tortoise and the hare.

2. Would it be helpful to try to put both graphs on the same pair of axes? Why or why not?

3. The tortoise and the hare start out together and after half an hour the hare stops to take a rest. How long does it take the tortoise to catch up?

2.4 A Car Wash

Here are two ways to represent a situation.

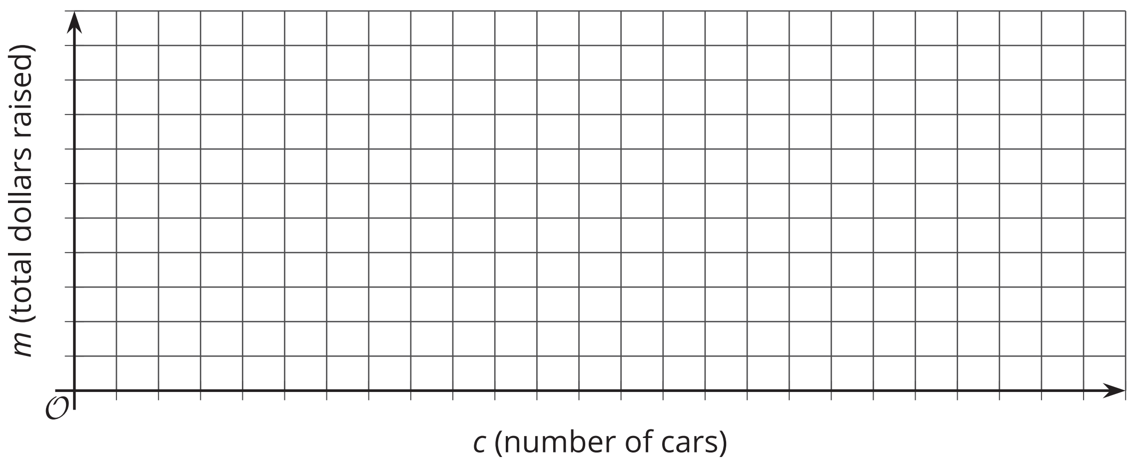
Description:

The Origami Club is doing a car wash fundraiser to raise money for a trip. They charge the same price for every car. After 11 cars, they raised a total of $93.50. After 23 cars, they raised a total of $195.50.

Table:

number of cars	amount raised in dollars
11	93.50
23	195.50

Create a graph that represents this situation.



2.5 Info Gap: Graphing Proportional Relationships

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me ________?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know ________ because . . . .”

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know ________?”
3. Listen to your partner’s reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Are you ready for more?

Ten people can dig 5 holes in 3 hours. If n people digging at the same rate dig m holes in d hours:

1. Is n proportional to m when $d = 3$?
2. Is n proportional to d when $m = 5$?
3. Is m proportional to d when $n = 10$?

Lesson 2 Summary

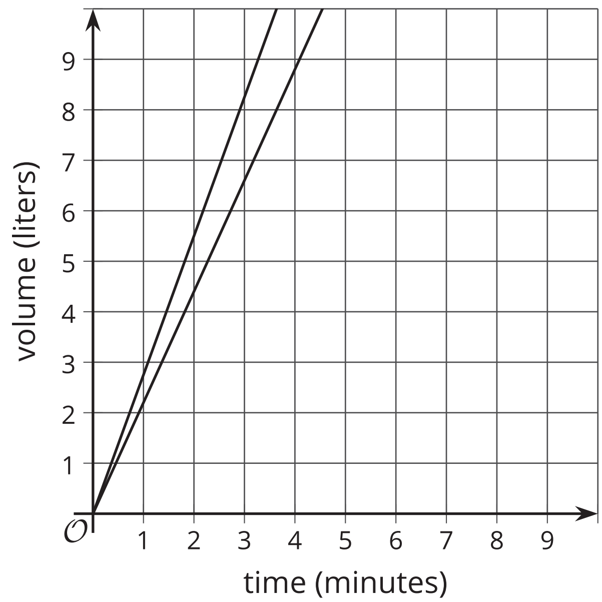
The scales we choose when graphing a relationship often depend on what information we want to know. For example, consider two water tanks filled at different constant rates.

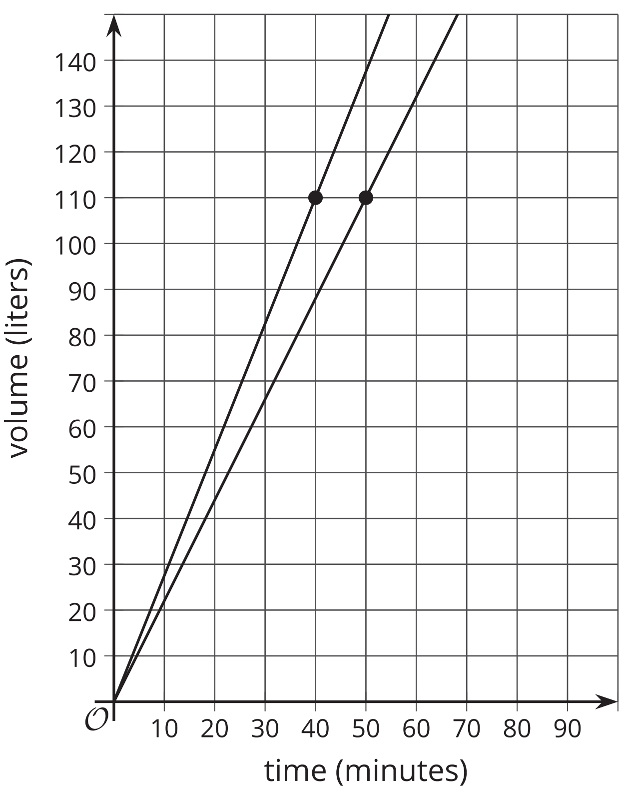
The relationship between time in minutes t and volume in liters v of Tank A can be described by the equation $v = 2.2t$.

For Tank B the relationship can be described by the equation $v = 2.75t$

These equations tell us that Tank A is being filled at a constant rate of 2.2 liters per minute and Tank B is being filled at a constant rate of 2.75 liters per minute.

If we want to use graphs to see at what times the two tanks will have 110 liters of water, then using an axis scale from 0 to 10, as shown here, isn't very helpful.





If we use a vertical scale that goes to 150 liters, a bit beyond the 110 we are looking for, and a horizontal scale that goes to 100 minutes, we get a much more useful set of axes for answering our question.

Now we can see that the two tanks will reach 110 liters 10 minutes apart—Tank B after 40 minutes of filling and Tank A after 50 minutes of filling.

It is important to note that both of these graphs are correct, but one uses a range of values that helps answer the question. In order to always pick a helpful scale, we should consider the situation and the questions asked about it.

What representation we choose for a proportional relationship also depends on our purpose. For example, if Tank C fills at a constant rate of 2.5 liters per minute and we want to see the change in volume every 30 minutes, we could use a table:

minutes (t)	liters (v)
0	0
30	75
60	150
90	225

No matter the representation or the scale used, the constant of proportionality is evident in each. In the equation, it is the number we multiply t by. In the graph it is the slope, and in the table it is the number by which we multiply values in the left column to get values in the right column. We can think of the constant of proportionality as a **rate of change**: the amount one variable changes by when the other variable increases by 1. For Tank A, the rate of change of v with respect to t is 2.2 liters per minute.