Proportional Relationships and Equations

Let's write equations describing proportional relationships.

1.1 Which Three Go Together: Expressions

Which three go together? Why do they go together?

A

$5 \cdot 2$

B

$4 \cdot ? = 20$

C

$x + 5$

D

$5x$

1.2 Feeding a Crowd, Revisited

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

   a. How many people will 1 cup of rice serve?

   b. How many people will 3 cups of rice serve? 12 cups? 43 cups?

   c. How many people will x cups of rice serve?

cups of dry rice	number of people
1	
2	6
3	
12	
43	
x	

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

   a. How many people will 1 spring roll serve?

   b. How many people will 10 spring rolls serve? 16 spring rolls? 25 spring rolls?

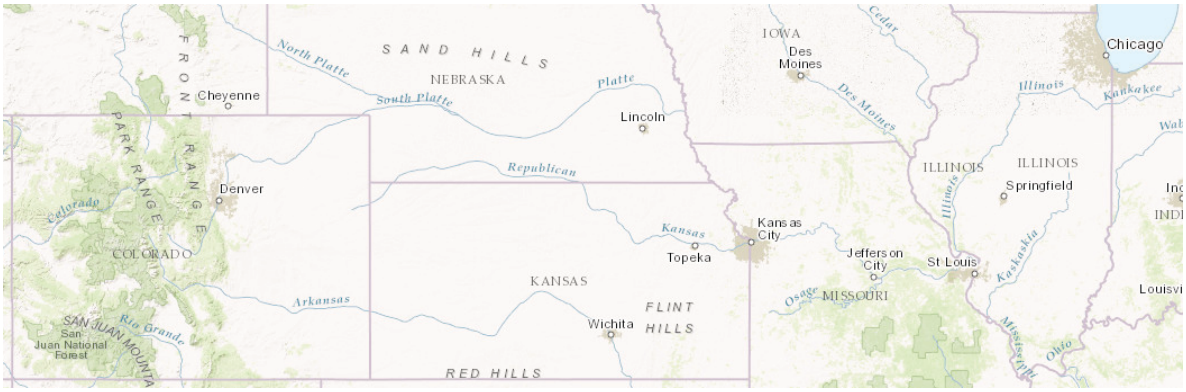
   c. How many people will n spring rolls serve?

number of spring rolls	number of people
1	
6	3
10	
16	
25	
n	

3. How was completing the table about spring rolls different from completing the table about rice? How was it the same?

1.3 Denver to Chicago

A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.



1. Complete the table.

time (hours)	distance (miles)
1	
1.5	915
2	
2.5	
t	

2. How far does the plane fly in 1 hour?
3. How far would the plane fly in t hours at this speed?

4. If d represents the distance that the plane flies at this speed for t hours, write an equation that relates t and d.

5. How far would the plane fly in 3 hours at this speed? in 3.5 hours? Explain or show your reasoning.

Are you ready for more?

A rocky planet orbits Proxima Centauri, a star that is about 1.3 parsecs from Earth. This planet is the closest planet outside of our solar system.

1. How long does it take light from Proxima Centauri to reach Earth? (A parsec is about 3.26 light years. A light year is the distance light travels in one year.)

2. Imagine there are two twins. One twin leaves Earth on a spaceship and travels to a planet near Proxima Centauri. The spaceship travels at 90% of the speed of light. The other twin stays home on Earth. How much does the twin on Earth age while the other twin travels to Proxima Centauri? (Do you think the answer would be the same for the twin on the spaceship? Consider researching "The Twin Paradox" to learn more.)

1.4 Revisiting Coco Bread

To bake coco bread, a bakery uses 200 milliliters of coconut milk for every 360 grams of flour. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of coconut milk to flour.

1. Complete the table.
2. Use f to represent the grams of flour needed for c milliliters of coconut milk. Write an equation that relates f and c.
3. How much flour is needed for 680 milliliters of coconut milk? 945 milliliters? Explain or show your reasoning.

coconut milk (milliliters)	flour (grams)
100	
200	360
450	
c	

Lesson 1 Summary

If the ratios between two corresponding quantities are always equivalent, the relationship between the quantities is called a **proportional relationship**. In this lesson, we wrote equations to represent proportional relationships described in words and shown in tables.

This table shows the amount of red paint and blue paint needed to make a certain shade of purple paint, called Venusian Sunset.

red paint (parts)	blue paint (parts)
3	12
1	4
7	28
$\frac{1}{4}$	1
r	$4r$

Note that "parts" can be *any* unit for volume. If we mix 3 cups of red with 12 cups of blue, we will get the same shade as if we mix 3 teaspoons of red with 12 teaspoons of blue.

The last row in the table shows that if we know the amount of red paint, r, we can always multiply it by 4 to find the amount of blue paint needed to make Venusian Sunset. If b is the amount of blue paint, we can say this more succinctly with the equation $b = 4r$. So, the amount of blue paint is proportional to the amount of red paint, and the constant of proportionality is 4.

We can also look at this relationship the other way around.

blue paint (parts)	red paint (parts)
12	3
4	1
28	7
1	$\frac{1}{4}$
b	$\frac{1}{4}b$

If we know the amount of blue paint, b, we can always multiply it by $\frac{1}{4}$ to find the amount of red paint, r, needed to make Venusian Sunset. So, the equation $r = \frac{1}{4}b$ also represents the relationship. The amount of red paint is proportional to the amount of blue paint, and the constant of proportionality $\frac{1}{4}$.

In general, when y is proportional to x, we can always multiply x by the same number k—the **constant of proportionality**—to get y. We can write this much more succinctly with the equation $y = kx$.