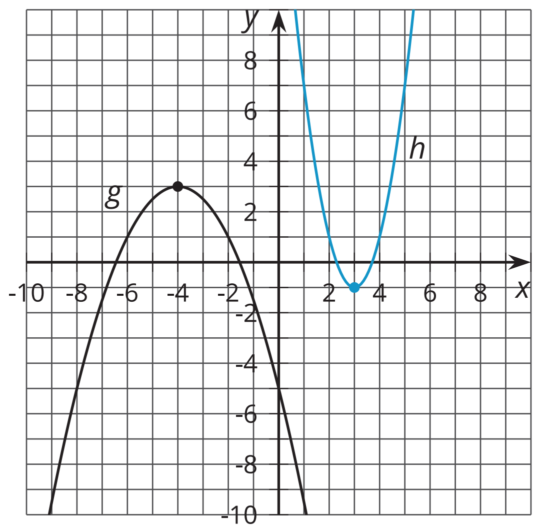
Unit 5, Lesson 13

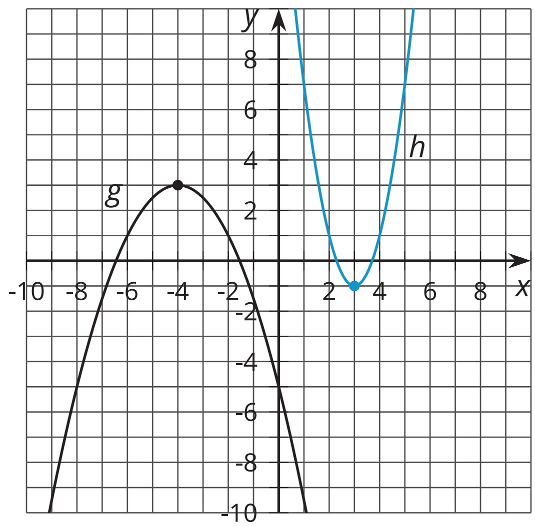
Transforming Parabolas

Let's look at transformations to parabolas.

13.1 Parabola Questions



13.2 The Form of a Transformation



1. What are the transformations from an original function $f(x) = x^2$ to g and to h?

2. Write an equation for g and for h using the transformations.

3. How does this equation compare to vertex form of a parabola? What features do you see in this form?

13.3 Find the Transformations

Your teacher will assign one of these equations to your group:

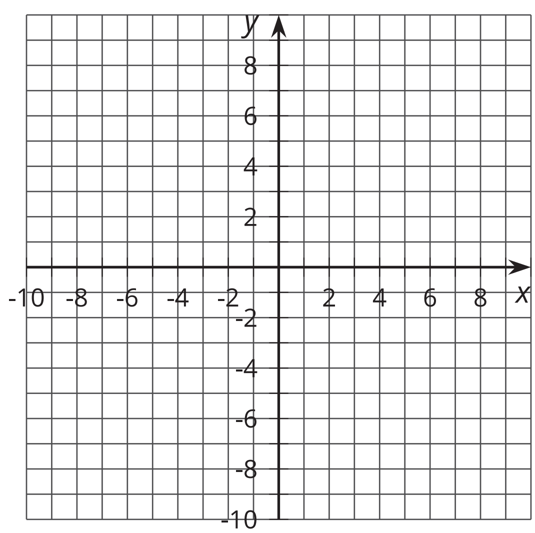
- $y = \frac{3}{4}x^2 - 6x + 17$
- $y = \frac{4}{3}x^2 + 8x + 6$
- $y = \frac{2}{3}x^2 - 4x - 3$

1. Rewrite your equation in vertex form by completing the square.

2. Identify the transformations from the equation $y = x^2$ to your equation.

3. Before graphing, identify the vertex and y-intercept.

4. Graph your equation.



Are you ready for more?

In Colombia, there is a popular game called *Rana*. Players toss coins into the mouth of a frog figurine to win points. Here is an equation for the path of the coin from the player's hand to the frog's mouth, in feet: $y = -\frac{3}{16}x^2 + \frac{3}{2}x$.

1. Rewrite the equation in vertex form.
2. If the player's hand is 2.5 feet above ground level, what is the height of the coin at its highest point?
3. The player's hand and the game board are at the same height. How far away is the player standing from the frog?

Lesson 13 Summary

When we have an equation for a parabola in vertex form, we can see the transformations from an original function $y = x^2$ without graphing. Here is an example:

The graph of $y = 4(x+6)^2 - 7$ has been shifted left 6, stretched vertically by a factor of 4, and shifted down 7. This makes sense because the original vertex is at $(0,0)$, and the new vertex is at $(-6,-7)$, so it has been shifted left 6 and down 7 as well.

We can also see the transformations from an equation that is not written in vertex form, but we will need to rewrite it first. Take a look at this equation: $y = \frac{4}{5}x^2 - 8x + 14$. Let's rewrite it in vertex form by completing the square:

$$\begin{aligned} y &= \frac{4}{5}x^2 - 8x + 14 \\ &= \frac{4}{5}(x^2 - 10x + \underline{\qquad\quad}) + 14 - \underline{\qquad\quad} \\ &= \frac{4}{5}(x^2 - 10x + 25) + 14 - 20 \\ &= \frac{4}{5}(x-5)^2 - 6 \end{aligned}$$

Now we can see that the vertex is at $(5,-6)$. Using this equation, we can identify the transformations from $y = x^2$: shift left 5, vertical stretch by a factor of $\frac{4}{5}$, shift down 6.

For any equation of a parabola in vertex form $y = a(x-h)^2 + k$, we can identify the transformations: horizontal translation by h, vertical stretch by a factor of a, reflection over the x-axis if $a < 0$, and vertical translation by k.