Unit 3, Lesson 5

Circumference and Wheels

Let's explore how far different wheels roll.

5.1 A Rope and a Wheel

Han says that you can wrap a 5-foot rope around a wheel with a 2-foot diameter because $\frac{5}{2}$ is less than pi. Do you agree with Han? Explain your reasoning.

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5.2 Rolling, Rolling, Rolling

Your teacher will give you a circular object.

1. Follow these instructions to create the drawing:
   a. On a separate piece of paper, use a ruler to draw a diagonal line all the way across the page.
   b. Roll your object along the line and mark where it completes one rotation.
   c. Use your object to draw tick marks along the line that are spaced as far apart as the diameter of your object.
2. What do you notice?

3. Use your ruler to measure these lengths to the nearest tenth of a centimeter:
   a. the diameter of your object
   b. how far your object rolled in one complete rotation
4. Find the quotient of how far your object rolled divided by its diameter. What do you notice?

5. If you wanted to mark where your object completes 2 rotations, how long a line would you need?

6. Compare your measurements and calculations with another group's that used a different object.
   a. What do you notice?

   b. If both groups rolled their object along the entire length of the classroom, which object would complete the most rotations? Explain or show your reasoning.

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5.3 Rotations and Distance

1. A car wheel has a diameter of 20.8 inches.

   a. About how far does the car wheel travel in 1 rotation? 5 rotations? 30 rotations?

   b. Write an equation relating the distance that the car travels in inches, c, to the number of wheel rotations, x.

   c. About how many rotations does the car wheel make when the car travels 1 mile? Explain or show your reasoning.

2. A bike wheel has a radius of 13 inches.

   a. About how far does the bike wheel travel in 1 rotation? 5 rotations? 30 rotations?

   b. Write an equation relating the distance that the bike travels in inches, b, to the number of wheel rotations, x.

   c. About how many rotations does the bike wheel make when the bike travels 1 mile? Explain or show your reasoning.

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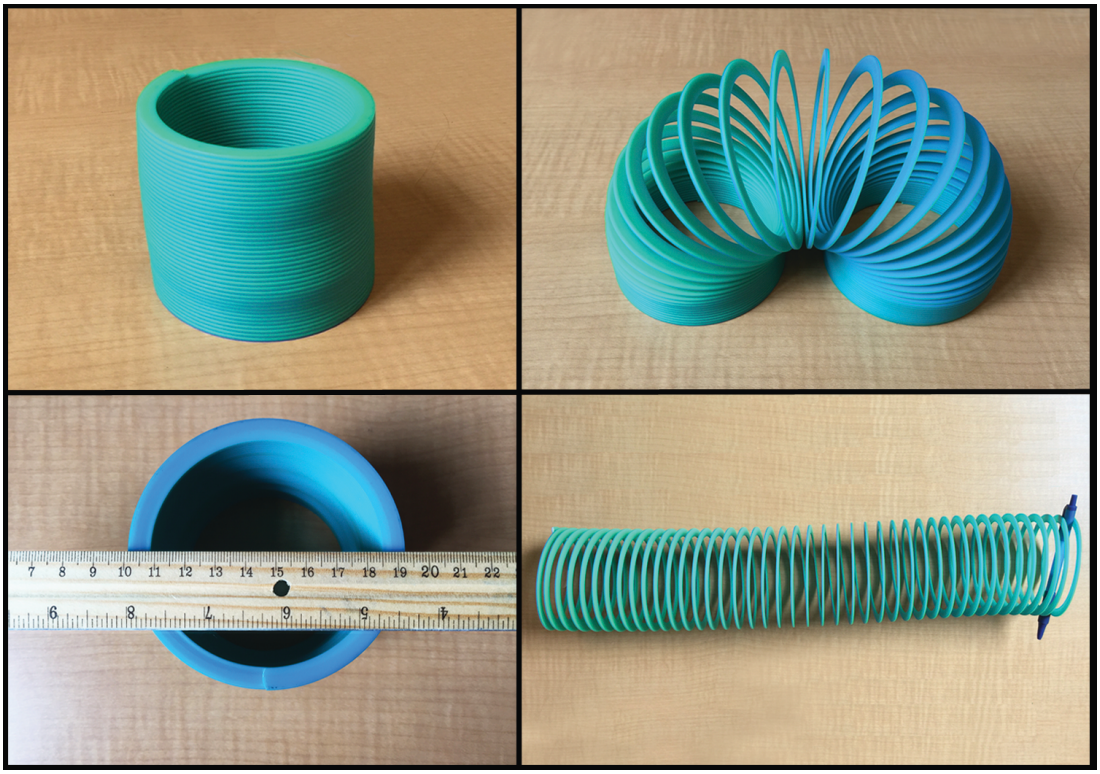
Are you ready for more?

Here are some photos of a spring toy.



If you could stretch out the spring completely straight, how long would it be? Explain or show your reasoning.

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5.4 Rotations and Speed

The circumference of a car wheel is about 65 inches.

1. If the car wheel rotates once per second, how far does the car travel in one minute?

2. If the car wheel rotates once per second, about how many miles does the car travel in one hour?

3. If the car wheel rotates 5 times per second, about how many miles does the car travel in one hour?

4. If the car is traveling 65 miles per hour, about how many times per second does the wheel rotate?

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Lesson 5 Summary

The circumference of a circle is the distance around the circle. This is also how far the circle rolls on flat ground in one rotation. For example, a bicycle wheel with a diameter of 2 feet has a circumference of 2π feet, which is about 6.3 feet. This means that the wheel will travel about 6.3 feet in one complete rotation.

We can use this relationship to calculate the distance traveled for any number of rotations. Here is a table showing approximately how far the bike travels when the wheel makes different numbers of rotations.

number of rotations	distance traveled in feet
1	6.3
2	12.6
3	18.9
10	63
50	315
x	$6.3x$

In the table, we see that the relationship between the distance traveled and the number of wheel rotations is a proportional relationship. The constant of proportionality is equal to the circumference of the wheel, 2π, or about 6.3.

For this wheel, the equation $d = 6.3x$ gives the distance traveled, d, when the wheel makes x rotations.

For a wheel of any size, the equation is $d = Cx$, where C is the circumference of the wheel.

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