Using Tables for Conditional Probability

Let's use tables to estimate conditional probabilities.

9.1 Math Talk: Fractions in Fractions

Find the value of each expression mentally.

- $\frac{7}{11} \div \frac{8}{11}$
- $\frac{\left(\frac{1}{3}\right)}{\left(\frac{2}{3}\right)}$
- $\frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}$
- $\frac{\left(\frac{3}{8}\right)\left(\frac{8}{19}\right)}{\left(\frac{5}{19}\right)}$

9.2 A Possible Cure

A pharmaceutical company is testing a new medicine for a disease, using 115 test subjects. Some of the test subjects are given the new medicine and others are given a placebo. The results of their tests are summarized in the table.

	no more symptoms	symptoms persist	total
given medicine	31	26	57
given placebo	16	42	58
total	47	68	115

1. Divide the value in each cell by the total number of test subjects to find each probability to two decimal places. Some of the values have been completed for you.

	no more symptoms	symptoms persist	total
given medicine	0.27		0.50
given placebo			
total			1

If one of these test subjects is selected at random, find each probability:

a. $P(\text{symptoms persist})$

b. $P(\text{given medicine and symptoms persist})$

c. $P(\text{given placebo or symptoms persist})$

2. From the original table, divide each cell by the total for the row to find the probabilities with row conditions. Some of the values have been completed for you.

	no more symptoms	symptoms persist	total
given medicine	0.54		
given placebo			1

a. $P(\text{symptoms persist} \mid \text{given medicine})$

b. $P(\text{no more symptoms} \mid \text{given placebo})$

3. Jada didn't read the instructions for the previous problem well and used the table she created on the first problem to divide each cell by the probability total for each row. For example, in the top left cell she calculated $0.27 \div 0.5$. Complete the table using Jada's method.

	no more symptoms	symptoms persist	total
given medicine			
given placebo			

What do you notice about this table?

4. From the original table, divide each cell by the total for the column to find the probabilities with column conditions. Some of the values have been completed for you.

	no more symptoms	symptoms persist
given medicine	0.66	
given placebo		
total		1

a. $P(\text{given medicine} \mid \text{symptoms persist})$

b. $P(\text{given placebo} \mid \text{no more symptoms})$

5. Are the events "symptoms persist" and "given medicine" independent events? Explain or show your reasoning.

6. Based on your work, does being given this medicine have an affect on whether symptoms persist or not?

Are you ready for more?

Consider the data collected from all students in grades 11 and 12 and their intentions of going to prom.

1. 65% of the students who are going to prom are from grade 12. Complete the table.

	going to prom	not going to prom	total
grade 11			127
grade 12			116
total	124	119	243

2. Based on your results, what is the probability that a student from this group, selected at random, is a student in grade 11 that did not go to prom?

9.3 The Blood Bank

A blood bank in a region has some information about the blood types of people in its community. Blood is grouped into types O, A, B, and AB. Each blood type either has the Rh factor (Rh+) or not (Rh-). If a person is randomly selected from the community, the probability of that person having each blood type and Rh factor combination is shown in the table.

	O	A	B	AB	total
Rh+	0.374	0.357	0.085	0.034	
Rh-	0.066	0.063	0.015	0.006	
total					1

1. What does the 0.085 in the table represent?

2. Use the table or create additional tables to find the probabilities, then describe the meaning of the event.

   a. $P(\text{O})$

   b. $P(\text{Rh+})$

   c. $P(\text{O and Rh+})$

   d. $P(\text{O or Rh+})$

   e. $P(\text{O} \mid \text{Rh+})$

   f. $P(\text{Rh+} \mid \text{O})$

Lesson 9 Summary

Organizing data in tables is a useful way to see information and compute probabilities. Consider the data collected from all students in grades 11 and 12 and their intentions of going to prom.

	going to prom	not going to prom	total
grade 11	43	84	127
grade 12	81	35	116
total	124	119	243

A student is randomly selected from this group of students. We can find a number of probabilities based on the table.

$P(\text{grade 12 and going to prom}) = \frac{81}{243}$ because there are 81 students who are both in 12th grade and going to prom out of all 243 students in the group who could be selected.

$P(\text{grade 12}) = \frac{116}{243}$ because there are 116 grade 12 students out of the entire group.

$P(\text{going to prom}) = \frac{124}{243}$ because there are 124 students going to prom out of the entire group.

$P(\text{grade 12 or going to prom}) = \frac{159}{243}$ since there are 159 students in grade 12 or going to prom (43 from grade 11 going to prom, 81 from grade 12 going to prom, and 35 from grade 12 not going to prom).

$P(\text{going to prom} \mid \text{grade 12}) = \frac{81}{116}$ represents the probability that the chosen student is going to prom under the condition that the student is in grade 12. The group we are considering is different for this probability because the condition is that the student is in grade 12. Imagine all grade 12 students are in a room and we are selecting from only this group. We see that 81 students from this group are going to prom out of the 116 students in the group.

$P(\text{grade 11} \mid \text{going to prom}) = \frac{43}{124}$ represents the probability that the chosen student is in grade 11 under the condition that the student is going to prom. In this case, we imagine that everyone going to prom is gathered in a room, and we are selecting one student from this group. The probability that this student is in grade 11 when considering only this group is $\frac{43}{124}$.

The last two probabilities can also be found using the multiplication rule.

$P(\text{A and B}) = P(\text{A} \mid \text{B}) \boldsymbol{\cdot} P(\text{B})$ can be rewritten $P(\text{A} \mid \text{B}) = \frac{P(\text{A and B})}{P(\text{B})}$.

For example, substituting values into $P(\text{going to prom} \mid \text{grade 12}) = \frac{P(\text{going to prom and grade 12})}{P(\text{grade 12})}$

gives $\frac{81}{116} = \frac{\left(\frac{81}{243}\right)}{\left(\frac{116}{243}\right)}$ which is a true equation.

