Combining Functions

Let's make some new functions using other functions.

10.1 Are Book Sales Improving?

t (years since 2010)	number of books sold in the US (millions)	population of the US (millions)
0	2,530	309.35
1	2,400	311.64
2	2,730	313.99
3	2,720	316.23
4	2,700	318.62
5	2,710	321.04
6	2,700	323.41

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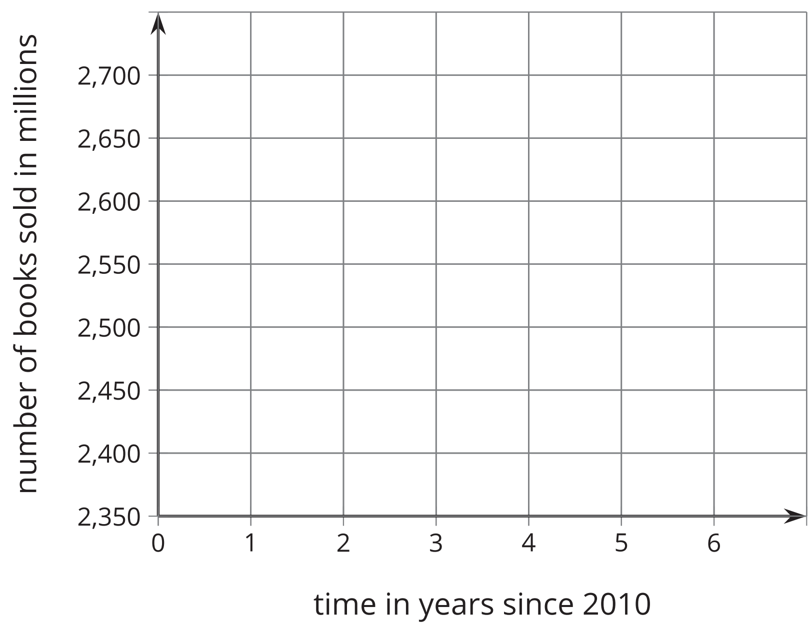
10.2 How Many Books Can One Person Have?

The table shows the values of two functions, P and B, where $P(t)$ is the population of the US, in millions, t years after 2010, and $B(t)$ is the number of books sold per year, in millions, t years after 2010.

t (years since 2010)	$B(t)$ (millions)	$P(t)$ (millions)	$R(t)$
0	2,530	309.35	
1	2,400	311.64	
2	2,730	313.99	
3	2,720	316.23	
4	2,700	318.62	
5	2,710	321.04	
6	2,700	323.41	

1. Plot the values of B as a function of t. What does the plot tell you about book sales?



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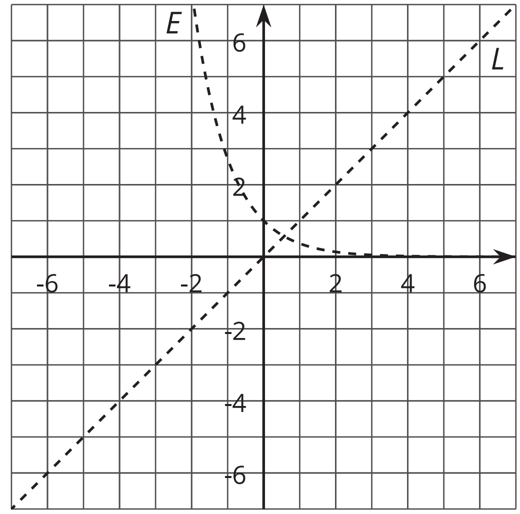
2. How many books were sold per person in 2010 and 2016? What do these values tell you about book sales?

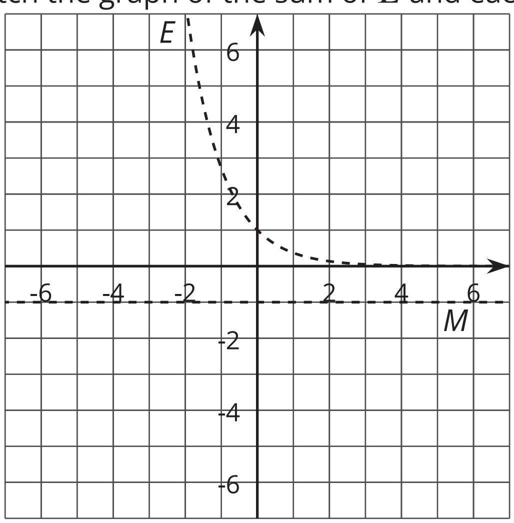
3. Define a new function R by $R(t) = \frac{B(t)}{P(t)}$. Complete the table and then graph the values of $R(t)$. What do the values of R tell you?

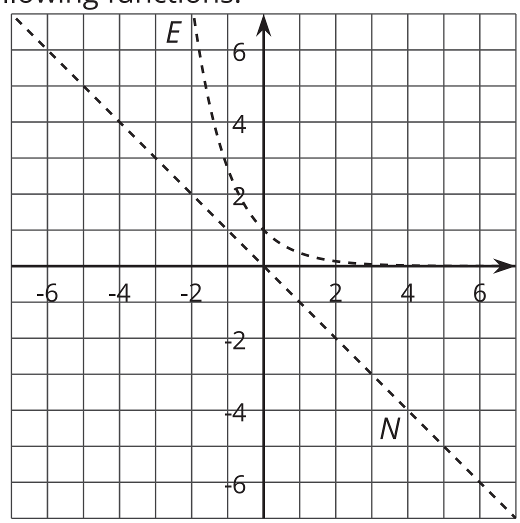
10.3 Adding Functions

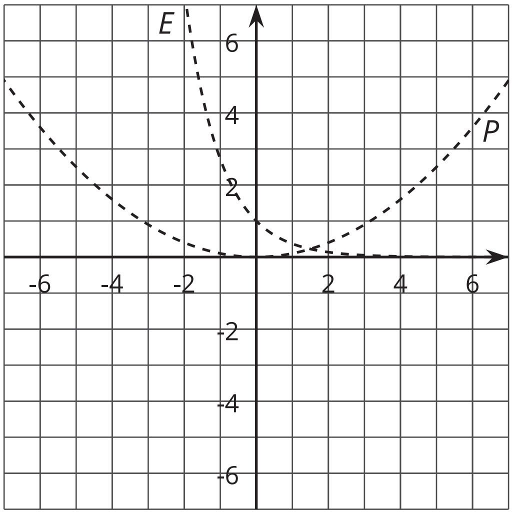
1. Here are the graphs of two functions, E and L. Define a new function S by adding E and L, so $S(x) = E(x) + L(x)$. On the same axes, sketch what you think the graph of S looks like.

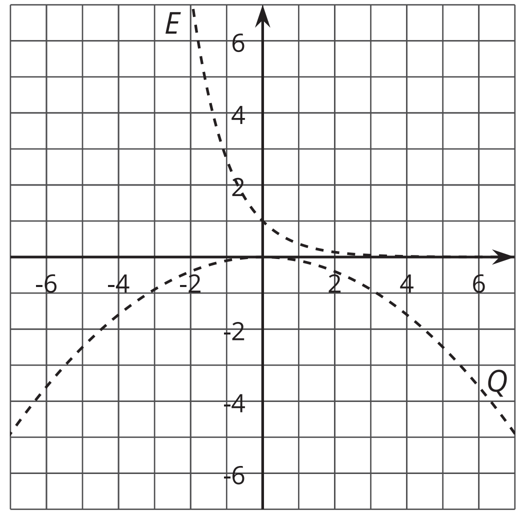


2. Sketch the graph of the sum of E and each of the following functions.



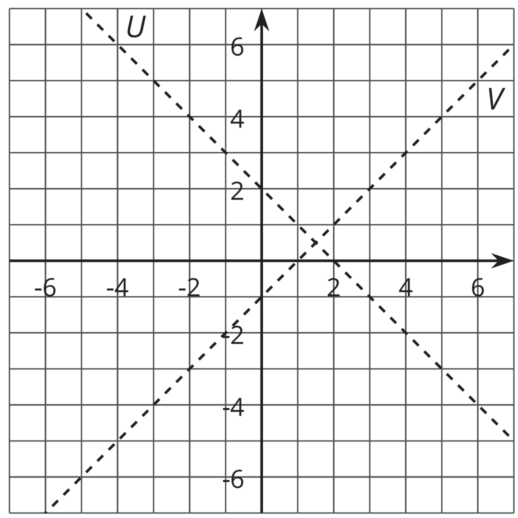






Are you ready for more?

Here are the graphs of two functions, U and V. Define a new function W by multiplying U and V, so $W(x) = U(x)V(x)$. On the same axes, sketch what you think the graph of W looks like.



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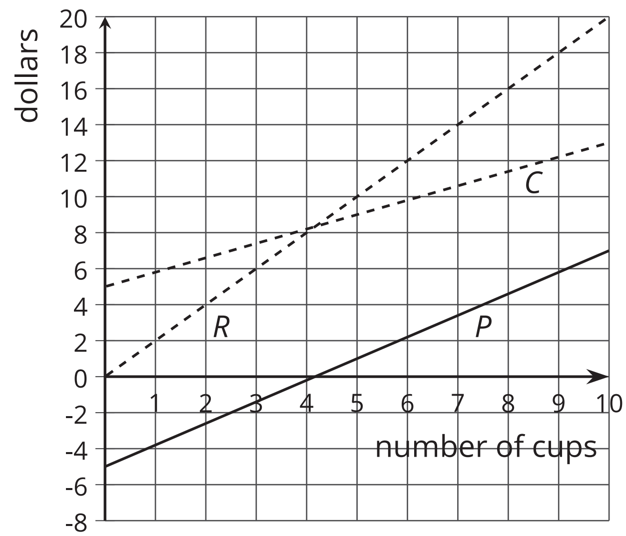
Lesson 10 Summary

We can add, subtract, multiply, and divide functions to get new functions. For example, the cost in dollars of producing n cups of lemonade at a lemonade stand is $C(n) = 5 + 0.8n$. The revenue (amount of money collected) from selling n cups is $R(n) = 2n$ dollars. The profit $P(n)$ from selling n cups is the revenue minus the cost, so

$$P(n) = R(n) - C(n) = 2n - (5 + 0.8n) = 1.2n - 5$$

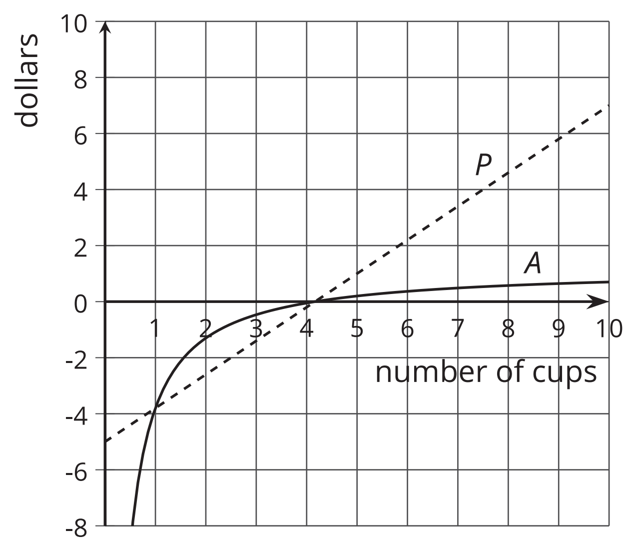
Here are the graphs of C, R, and P. Can you see how each value on P is the result of the difference between the corresponding points on R and C?



The average profit per cup, $A(n)$, from selling n cups, is the quotient of the profit and the number of cups, so

$$A(n) = \frac{P(n)}{n} = \frac{1.2n - 5}{n} = 1.2 - \frac{5}{n}$$

Here are the graphs of P and A. Can you see how the value of $A(n)$ is the result of the quotient of $P(n)$ and n? Why does it make sense that both functions are negative when $n < 4\frac{1}{6}$ and positive when $n > 4\frac{1}{6}$?



Since n can only be positive, $P(n)$ and $A(n)$ always have the same sign for a given n value. Notice that for the average profit to be positive, the seller has to sell at least 5 cups (since $4\frac{1}{6}$ is not in the domain, we must round up). It is also true that for a large number of cups, the average profit is close to \$1.20 per cup.

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