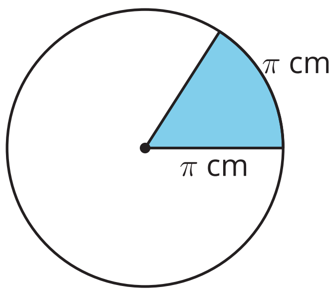
Radian Sense

Let's get a sense for the sizes of angles measured in radians.

16.1 Which Three Go Together: Angle Measures

Which three go together? Why do they go together?

A



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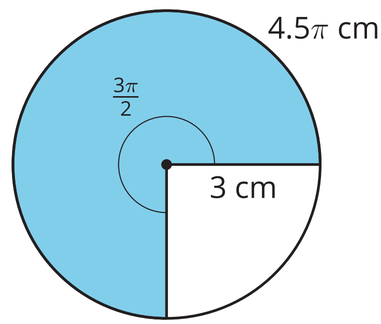

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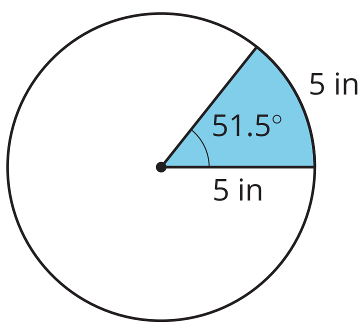

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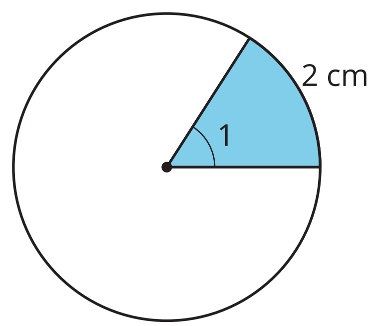
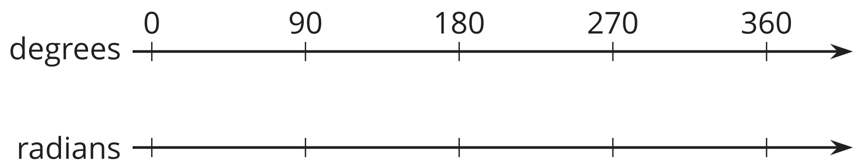


16.2 Degrees versus Radians

This double number line shows measures in degrees on one line and in radians on another.



1. Fill in the radian measures on the bottom line for 0°, 90°, 180°, 270°, and 360°.
2. Express each radian measurement in degrees.
   a. $\frac{\pi}{3}$ radians
   b. $\frac{5\pi}{4}$ radians
3. Express each degree measurement in radians.
   a. 30°
   b. 120°

Are you ready for more?

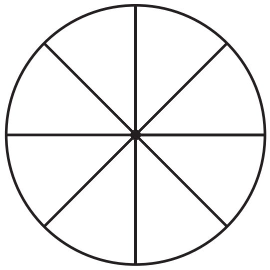
Your boat is heading due south when you hear that you must head northeast to return home. You turn the boat, traveling in a counterclockwise circular path until you're facing northeast.

1. Sketch the path of the boat.
2. If the circle you traced had a radius of 40 feet, what distance did you travel?

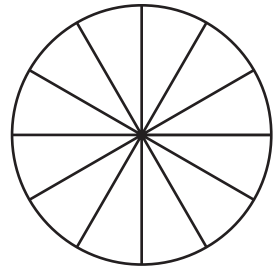
16.3 Pie Coloring Contest

Your teacher will give you a set of cards with angle measures on them. Place the cards upside down in a pile. Take turns with your partner drawing a card.

1. For each card you draw, shade a sector on one of the circles whose central angle is the measure on the card. If you are shading in a circle that already has a shaded sector, choose a spot next to an already-shaded sector—don't leave any gaps. You might have to draw additional lines to break the existing sectors into smaller pieces. Explain to your partner how you knew how much to shade, then complete a row of the table that corresponds to the circle.
2. For each card your partner draws, listen carefully to their explanation for how much they shaded. If you disagree, discuss your thinking and work together to reach an agreement.
3. Continue until you pull a card with an angle measure that won't fit in any of the sectors that are still blank.



card	measure in radians (may be blank)	total shaded, in radians



card	measure in radians (may be blank)	total shaded, in radians

When you’re finished, answer these questions about each circle:

1. What is the central angle measure for the remaining unshaded sector in both degrees and radians?
2. What is the central angle measure for the block of shaded sectors in both degrees and radians?

16.4 What Fraction?

For each angle, you have been given a radian measure, a fraction of a circle, or a degree measure. Complete the table with the missing information.

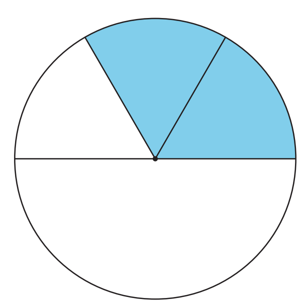
degrees	fraction of a circle	radians
90	$\frac{1}{4}$	$\frac{\pi}{2}$
120		
	$\frac{1}{24}$	
		$\frac{11\pi}{12}$
		$\frac{13\pi}{12}$
	$\frac{2}{3}$	
		$\frac{\pi}{6}$
315		
	$\frac{11}{12}$	
0		
	$\frac{3}{8}$	
		$\frac{5\pi}{3}$
210		

degrees	fraction of a circle	radians
45		
		2π
255		
		$\frac{23\pi}{12}$
	$\frac{1}{6}$	
150		
		$\frac{7\pi}{12}$
	$\frac{19}{24}$	
		$\frac{5\pi}{12}$
		$\frac{3\pi}{2}$
	$\frac{5}{8}$	
	$\frac{1}{2}$	

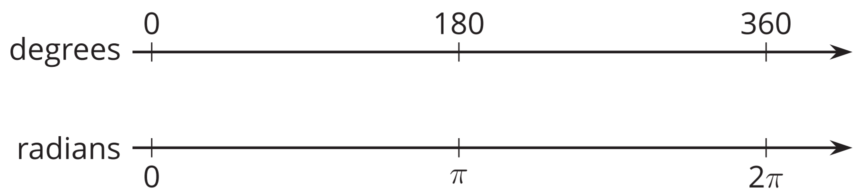
Lesson 16 Summary

We can divide circles into congruent sectors to get a sense for the size of an angle measured in radians.

Suppose we want to draw an angle that measures $\frac{2\pi}{3}$ radians. We know that π radians is equivalent to 180 degrees. If we divide a sector with a central angle of π radians into thirds, we can shade in two of the sectors to create an angle measuring $\frac{2\pi}{3}$ radians.



Another way to understand the size of an angle measured in radians is to create a double number line with degrees on one line and radians on the other. On the double number line shown here, the degree measures are aligned with their equivalent radian measures. For example, π radians is equivalent to 180˚.



Suppose we need to know the size of an angle that measures $\frac{3\pi}{4}$ radians. The left half of the double number line represents π radians. Divide the left half of the top and bottom number lines into fourths, then count out 3 of them on the radians line to land on $\frac{3\pi}{4}$. On the top line, each interval we drew represents 45 degrees because $180 \div 4 = 45$. If we count 3 of those intervals, we find that $\frac{3\pi}{4}$ radians is equivalent to 135 degrees because $45 \cdot 3 = 135$.