Unit 6, Lesson 14

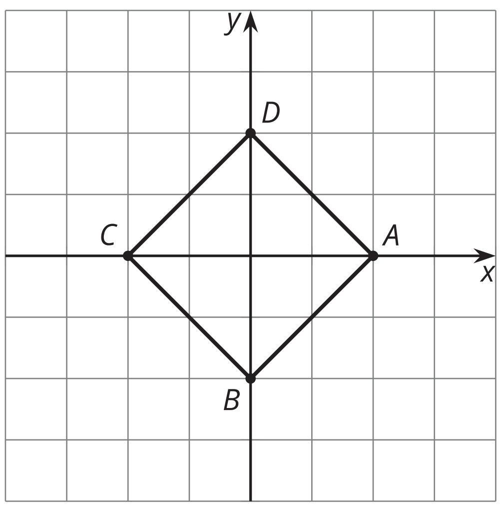
Coordinate Proof

Let’s use coordinates to prove theorems and to compute perimeter and area.

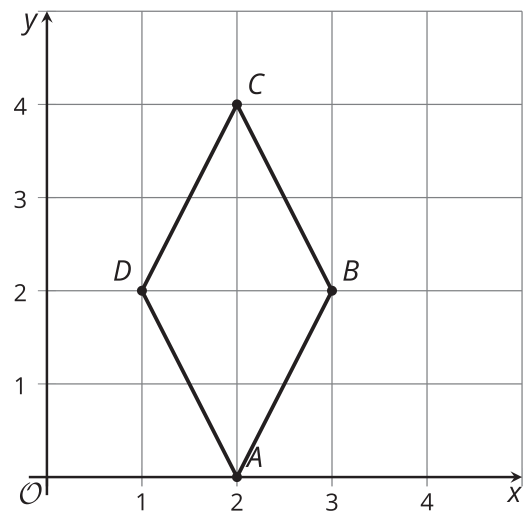
14.1 Which Three Go Together: Coordinate Quadrilaterals

Which three go together? Why do they go together?

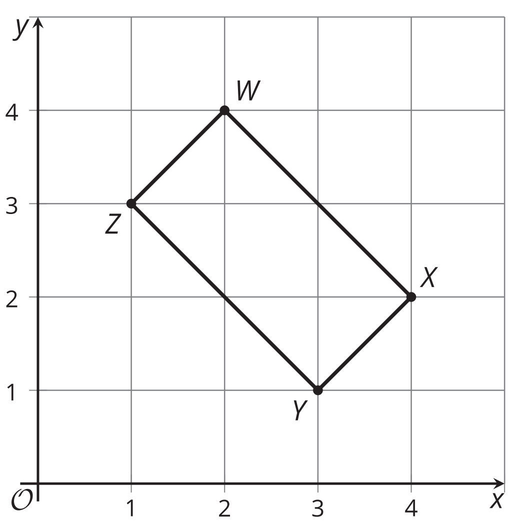
A



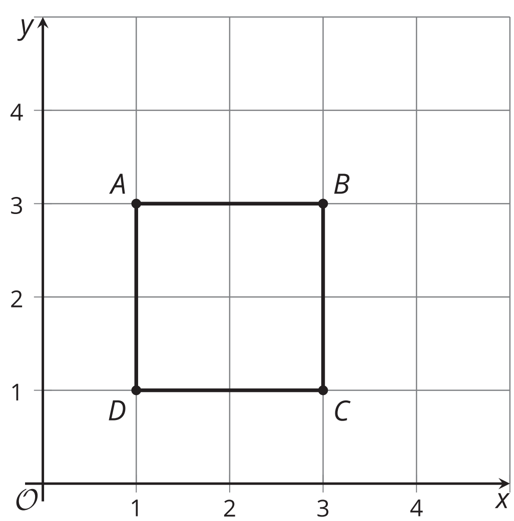
B



C



D



14.2 Name This Quadrilateral

A quadrilateral has vertices $(0, 0)$, $(4, 3)$, $(13, -9)$, and $(9, -12)$.

1. What type of quadrilateral is it? Explain or show your reasoning.

2. Find the perimeter of this quadrilateral.

3. Find the area of this quadrilateral.

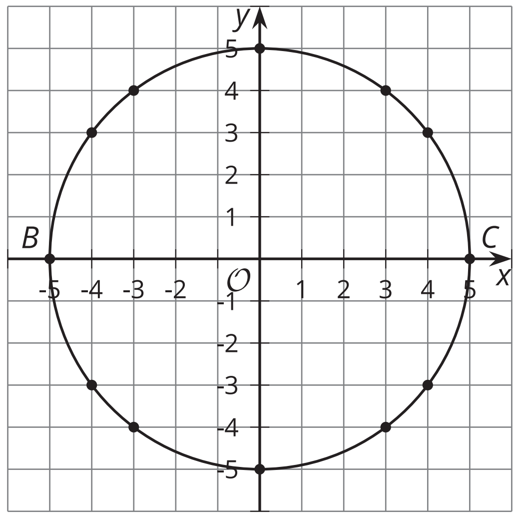
Are you ready for more?

1. A parallelogram has vertices $(0, 0)$, $(5, 0)$, $(-2, 10)$, and $(3, 10)$. Find the area of this parallelogram.

2. Consider a general parallelogram with vertices $(0, 0)$, (a, b), (kb, ka), and $(a - kb, b + ka)$, where a and b are positive, and a scale factor of k. Show that the parallelogram is a rectangle, then write an expression for its area in terms of a, b, and k.

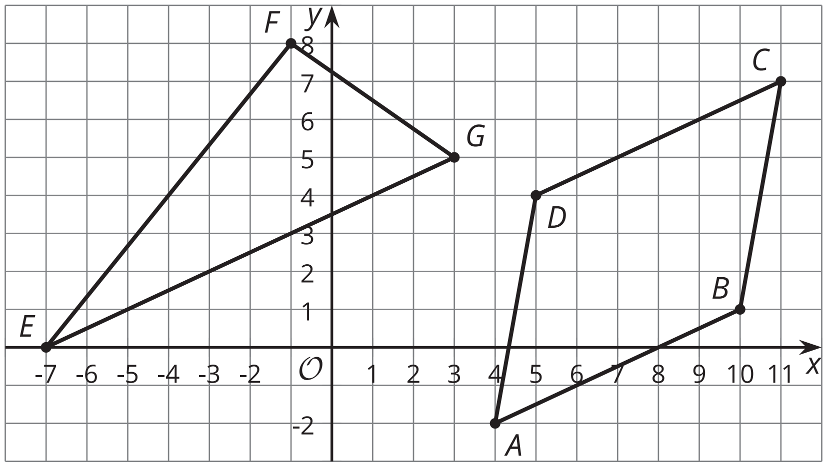
14.3 Circular Logic

The image shows a circle with several points plotted on the circle.



1. How does segment BC relate to the circle?
2. Choose one of the plotted points on the circle and call it D. Each student in the group should choose a different point. Draw segments BD and DC. What does the measure of angle BDC appear to be?
3. Calculate the slopes of segments BD and DC. What do your results tell you?
4. Compare your results to those of others in your group. What did they find?
5. Using your group's results, write a conjecture that captures what you are seeing.

Lesson 14 Summary



What kind of shape is quadrilateral $ABCD$? It looks like it might be a rhombus. To check, we can calculate the length of each side. Using the Pythagorean Theorem, we find that the lengths of segments AB and CD are $\sqrt{45}$ units, and the lengths of segments BC and DA are $\sqrt{37}$ units. All side lengths are between 6 and 7 units long, but they are not exactly the same. So our calculations show that $ABCD$ is not really a rhombus, even though at first glance we might think it is.

We did just show that two pairs of opposite sides of $ABCD$ are congruent. This means that $ABCD$ must be a parallelogram. Checking slopes confirms this. Sides AB and CD both have a slope of $\frac{1}{2}$. Sides BC and DA both have a slope of 6.

Can we find the area of triangle EFG? That seems tricky, because we don't know the height of the triangle using EG as the base. However, angle EFG seems like it could be a right angle. In that case, we could use sides EF and FG as the base and height.

To see if EFG is a right angle, we can calculate slopes. The slope of EF is $\frac{8}{6}$ or $\frac{4}{3}$, and the slope of FG is -$\frac{3}{4}$. Since the slopes are opposite reciprocals, the segments are perpendicular and angle EFG is indeed a right angle. This means that we can think of EF as the base and FG as the height. The length of EF is 10 units and the length of FG is 5 units. So the area of triangle EFG is 25 square units because $\frac{1}{2} \cdot 10 \cdot 5 = 25$.